

## ABSTRACTS

DAY 1: SATURDAY, MARCH 29

### **Katie Anders, 8:55-9:55**

*Title:* Hyper- $d$ -ary and balanced  $d$ -ary representations for even bases  $d$

*Abstract:* We first consider hyperbinary representations and balanced binary representations and their connection to the Stern sequence. We then explore hyperquaternary and balanced quaternary representations and see the Fibonacci numbers make an appearance. We conclude by briefly considering how our results extend to higher even bases.

*Room A*

### **Ayla Gafni, 9:55-10:20**

*Title:* Exponential Sums with Additive Coefficients

*Abstract:* For an arithmetic function  $f$  and a real number  $\alpha$ , consider the exponential sum

$$S_f(x, \alpha) = \sum_{n \leq x} f(n) e^{2\pi i n \alpha}.$$

The growth of these sums as  $x$  increases plays an important role in many number theory techniques. We will discuss new bounds on these exponential sums for various additive functions  $f$ , including  $\omega(n)$  (the number of distinct prime factors of  $n$ ) and  $\Omega(n)$  (the total number of prime factors of  $n$ ). We will then apply these bounds to enumerate certain integer partitions and solutions to Diophantine equations. This is joint work with Nicolas Robles.

### **Yaghoub Rahimi, 10:20-10:45**

*Title:* A Density Theorem for Higher Order Sums of Prime Numbers

*Abstract:* "Let  $P$  be a subset of the primes of lower density strictly larger than  $\frac{1}{2}$ . Then, every sufficiently large even integer is a sum of four primes from the set  $P$ . We establish similar results for  $k$ -summands, with  $k \geq 4$ , and for  $k \geq 4$  distinct subsets of primes. This extends the work of H. Li, H. Pan, as well as X. Shao on sums of three primes, and A. Alsteri and X. Shao on sums of two primes. The primary new contributions come from elementary combinatorial lemmas. "

### **Santiago Radi, 11:15-11:40**

*Title:* Applications of the fixed-point proportion to Arithmetic Dynamics

*Abstract:* Given a sequence of numbers in a number field, can we estimate the density of prime ideals that divide at least one term in the sequence? Among all the different kinds of sequences we may take, we assume the sequence is given by the orbit of a polynomial. This for example includes Fermat's numbers, Sylvester's sequence or Fibonacci's sequence.

Using Galois theory, the problem can be restated as a problem of groups acting on trees, where the aforementioned density is bounded by the fixed-point proportion, a quantity that measures the proportion of elements in the group fixing at least one vertex in each level of the tree.

In this talk, I will present two results. The first one, in collaboration with Jorge Fariña-Asategui, shows that in a certain family of groups (known as super strongly fractal groups), the fixed-point proportion is zero. This allowed us to find explicit examples to a question of Rafe Jones about the fixed-point proportion of dynamically exceptional polynomials.

If time permits, the second result is about the construction of a family of groups whose fixed-point proportion is non-zero, which as a consequence, it allows us to calculate the fixed-point proportion associated to the polynomials  $x^d + 1$  over the rational numbers.

*Room B***Avi Mukhopadhyay, 9:30-9:55***Title:* Mock Theta Transformations without Watson

*Abstract:* Ramanujan in his last letter to G.H. Hardy introduced mock theta functions, provided examples and various relations between them. G.N. Watson found transformations for the third order mock theta functions  $f(q)$  and  $\omega(q)$ . Zwegers in 2000 built on Watson's techniques to complete these mock theta functions and connected them to real analytic modular forms. We show how to derive these transformations using Lerch sums. To show the equivalence of the results involves some new  $q$ -series identities thus resulting in a new proof of Zwegers' theorem. This is joint work with Dr. Frank Garvan.

**Kalani Thalagoda, 9:55-10:20***Title:* Summation formula for mock modular forms

*Abstract:* Fourier coefficients of mock modular forms may behave erratically but the weighted sums might have much nicer behavior. One way to understand this phenomenon is by obtaining summation formulas. In this talk I will discuss such a summation formula for mock modular form with moderate growth. This is joint work with Beckwith, Gupta, Diamantis, Rolén.

**Jiaqi hou, 11:05-11:30***Title:* Kakeya-Nikodym norms of Maass forms

*Abstract:* I will talk about the problems on bounding  $L^p$  norms of Laplace eigenfunctions on Riemannian manifolds, and, in particular, Maass forms on locally symmetric spaces. The classical result was proved by Sogge, which is sharp on spheres but is expected to be improved with some curvature assumption. When  $p$  is small, Blair and Sogge showed that the problem can be reduced to studying the Kakeya-Nikodym norms, which measure the concentration near geodesic tubes. Using number theory method, we can prove some power savings for Kakeya-Nikodym norms of Hecke-Maass forms in some cases. In this talk, I will present the results on  $SL(2, \mathbb{C})$  and  $U(2, 1)$ .

**Edna Jones, 11:30-12:30***Title:* On the local-global conjecture for 3-dimensional Kleinian sphere packings

*Abstract:* We will discuss the local-global conjecture for certain integral Kleinian sphere packings, such as Soddy sphere packings and orthoplicial Apollonian sphere packings. Sometimes each sphere in a Kleinian sphere packing has a bend ( $1/\text{radius}$ ) that is an integer. When all the bends are integral, which integers appear as bends? In 2019, Kontorovich proved the local-global conjecture for Soddy sphere packings. Work towards proving a local-global conjecture for orthoplicial Apollonian sphere packings has been done by Dias and Nakamura. We extend their work to say more about the local-global conjecture for orthoplicial Apollonian sphere packings.

*Room A***Nathan Green, 2:05-2:30***Title:* Multiple Polylogarithm Relations in Characteristic  $p$ 

*Abstract:* An ongoing problem in the study of multiple zeta values (MZV) is to describe (and prove) the set of all linear relations between MZV. For real valued MZV, we have a clear conjecture about the origin of all linear relations and how many there are, but we are very far from proving it. For characteristic- $p$  valued (function field) MZV, we have already proved dimensions of the sets of relations, but we still don't have a full understanding of what these relations are! In my talk I will explain the basics of this theory and discuss recent advances I have made (joint with Federico Pellarin) in describing families of relations.

**J. C. Saunders, 2:30-2:55**

*Title:* Combinations of Cranks Modulo 15

*Abstract:* In this talk, we express the generating function of a combination of cranks of partitions that belong to congruence classes 3, 4, 5, 6 and 7 modulo 15 in terms of Eisenstein series. Our results rely on the observation that the generating function of the combination we consider is a weight 1 modular form on  $\Gamma_0(225)$  with the non-trivial modulo 3 primitive Dirichlet character. We obtain the modularity result by analyzing the transformation properties of related Klein forms. This is joint work with Dr. Selcuk Aygin and Dr. Khoa Nguyen.

**Jena Gregory, 2:55-3:20**

*Title:* Combinatorial statistics witnessing an infinite family of congruences for a sum of partition functions

*Abstract:* In 2007, Kronholm established infinite families of congruences in arithmetic progression, modulo any prime  $\ell$ , for  $p(n, m)$ , the function enumerating the partitions of  $n$  into parts whose sizes come from the set  $[m]$ . In 2022, Eichhorn, Kronholm, and Larsen proved there are combinatorial statistics, known as cranks, that witness Kronholm's infinite families of congruences.

In this talk, we explore an extension of these results and consider cranks witnessing a sum/difference congruence of the form  $p(n, m) \pm p(n', m) \equiv 0 \pmod{\ell}$ , where  $n'$  is determined by  $n$ .

By an analysis of Ehrhart's  $h^*$ -vector, we have established that for certain primes and small values of  $m$ , there are cranks witnessing this sum/difference congruence.

**Haouchen Wu, 3:55-4:20**

*Title:* Generalized Gauss Composition and Orthogonal Modular Forms for Binary Quadratic Forms

*Abstract:* We generalize Gauss's composition law for binary quadratic modules over Dedekind domains in a categorical way via the even Clifford algebra. This allows us to classify the class sets of different types of genera in terms of the class groups of quadratic algebras. We also associate orthogonal modular forms arising from positive definite binary lattices over the ring of integers of a totally real number field to Hilbert modular forms. This is joint work with John Voight.

**Zhumagali Shomanov, 4:20-4:45**

*Title:* A simple proof of the Atkin-O'Brien partition congruence conjecture for powers of 13

*Abstract:* In 1967, Atkin and O'Brien conjectured congruences for the partition function involving Hecke operators modulo powers of 13. In this paper we offer a simple proof of this conjecture.

*Room B*

**Koustav Mondal, 1:40-2:05**

*Title:* Relating Elliptic Curve point counting and coefficients of congruent theta series

*Abstract:* Eisenstein series plays a crucial role in many parts of number theory. In this talk, we will explore a method for constructing a basis for the space of Eisenstein series using specialized level-raising operators. As an application, we obtain the relations between the coefficients of congruent theta series and point counts on elliptic curves over finite fields.

**Paresh Singh Arora, 2:05-2:30**

*Title:* Well-poised Hypergeometric functions

*Abstract:* In this talk, we will present a detailed decomposition identity for the  ${}_3F_2$  hypergeometric function evaluated at  $-1$ , for cases where the parameters exhibit an additional symmetry known as "well-poisedness," using a Galois-theoretic approach. We will explore specific examples where the corresponding Galois representation contains a modular component. If time permits, we will extend our discussion to the case of  ${}_5F_4$  evaluated at  $-1$ .

**Pengcheng Zhang, 2:30-2:55**

*Title:* Ordinary primes of weight 2 modular forms

*Abstract:* A normalized Hecke eigenform is called  $p$ -ordinary for a prime  $p$  not dividing the level if its Hecke eigenvalue at  $p$  is a  $p$ -adic unit. It is expected that the set of ordinary primes of any newform has positive density. In a joint work with Tian Wang, we show that this is true for any weight 2 newform with coefficient field of degree  $q$  or  $2q$  for some prime  $q$ . The proof is carried out in the general setting of Galois representations, and the same technique can be used to show that the set of ordinary primes of certain  $GL_2$ -type abelian variety, including all  $GL_2$ -type abelian varieties over  $\mathbb{Q}$  of dimension  $q$  or  $2q$  for some prime  $q$ , has positive density.

**Mohit Tripathi, 2:55-3:20**

*Title:* Hypergeometric functions and modular forms

*Abstract:* In this talk, we will explore the definition of hypergeometric functions in finite fields,  $p$ -adic, and classical settings. We will present new summation formulas for  $p$ -adic hypergeometric functions and establish new relations between the Fourier coefficients of modular forms and  $p$ -adic hypergeometric functions. This is joint work with Dermot McCarthy.

**Akio Nakagawa, 3:55-4:20**

*Title:* Confluent hypergeometric functions over finite fields

*Abstract:* Recently, Otsubo redefined hypergeometric functions over finite fields. This definition is most convenient to treat confluent hypergeometric functions over finite fields. In this talk, I will introduce about Otsubo's definition, properties of confluent hypergeometric functions, their applications and relation with algebraic varieties as time permits.

**Hasan Saad, 4:20-4:45**

*Title:* Hypergeometric Distributions and Joint Families of Elliptic Curves

*Abstract:* In the 1960's, motivated by the Sato–Tate conjecture, Birch proved that the traces of Frobenius of elliptic curves over large finite fields is modeled by the semicircular distribution. Inspired by Birch's result, in recent work, Ono, Saikia and Saad studied the distribution of finite field hypergeometric functions that arise from Frobenius traces and symmetric squares of Frobenius traces for some families of elliptic curves using modular forms and harmonic Maass forms. In this talk, we explore Sato–Tate type distributions for some higher-order finite field hypergeometric functions that relate to joint families of elliptic curves. We also elucidate the previous results in a more geometric perspective.

**Frank Garvan, 4:45-5:45**

*Title:* Mock Atkin-Lehner Symmetry

*Abstract:* At the 1987 Ramanujan Centenary Meeting, Dyson predicted Mock-Atkin Lehner Symmetry. We describe Morris Newman's approach to  $1/2$  integer weight Hecke operators using trace operators and apply this approach to two of Ramanujan's third order mock theta functions. We derive some surprising congruences by using a form of Atkin-Lehner symmetry. These congruences are related to earlier work by Bruinier and Ono on Borcherds products.

## 1. DAY 2: SUNDAY, MARCH 30TH

**8:55-9:55, Samit Dasgupta**

*Title:* Brumer-Stark units and Explicit Class Field Theory

*Abstract:* We survey our recent work with Mahesh Kakde on the Brumer-Stark conjecture and the construction of class fields of totally real fields.

*Room A***Bella Tobin, 9:55-10:20***Title:* Points on the  $p$ -adic degree  $d$  Mandelbrot set*Abstract:* There is a natural extension of the Mandelbrot set to the  $p$ -adic setting and for degree  $d$  polynomials, however very little is known about this set. We establish a point on the  $p$ -adic degree  $d$  Mandelbrot set for certain pairs  $(d, p)$ .**Wade Hindes, 10:20-10:55***Title:* Preperiodic points for  $x^d + c$  in large degree*Abstract:* Let  $K$  be a number field. Then we completely classify the preperiodic portraits of the maps  $f(x) = x^d + c$  where  $c \in K$  is an algebraic integer and  $d \gg_K 0$ . More precisely, we prove that, up to accounting for the natural action of  $d$ th roots of unity on the preperiodic points of  $f$ , there are exactly thirteen such portraits up to isomorphism. The main tool we use is Baker's method on linear forms in logarithms. Moreover, we achieve the same classification for all  $c \in K$  (not necessarily integral) assuming the abc-conjecture and unconditionally for function fields. Finally, we apply this work to construct many irreducible polynomials with large dynamical Galois groups in semigroups generated by sets of polynomials of the form  $x^d + c$  under composition.**Alex Rice, 11:05-11:30***Title:* Notes and Computations on Forbidden Differences*Abstract:* Dating at least to the 1970s, a popular family of questions and results in arithmetic combinatorics concerns the existence of certain differences in dense sets of integers. The most famous results are due to Sárközy, who showed that a subset of  $\{1, 2, \dots, N\}$  with no perfect square differences has size  $o(N)$ , along with the analogous fact for differences that are one less than a prime number. In work stemming from a Summer 2024 undergraduate research program, we approach some related, more elementary questions, including exact formulas for sets lacking prime or shifted square differences, and exact threshold values for sets lacking square and  $p - 1$  differences for  $N$  up to 300 and 500, respectively. The latter computations were executed by leveraging existing algorithms for finding maximum cliques in graphs.*Room B***Pan Yan, 9:55-10:20***Title:*  $L$ -functions for  $\mathrm{Sp}(2n) \times \mathrm{GL}(k)$  via non-unique models*Abstract:* Let  $n$  and  $k$  be positive integers such that  $n$  is even. We derive new global integrals for  $\mathrm{Sp}(2n) \times \mathrm{GL}(k)$  from the generalized doubling method of Cai, Friedberg, Ginzburg and Kaplan, following a strategy and extending a previous result of Ginzburg and Soudry on the case  $n = k = 2$ . We show that these new integrals unfold to non-unique models on  $\mathrm{Sp}(2n)$ . Using the New Way method of Piatetski-Shapiro and Rallis, we show that these new global integrals represent the  $L$ -functions for  $\mathrm{Sp}(2n) \times \mathrm{GL}(k)$ , generalizing a previous work of Piatetski-Shapiro and Rallis on  $\mathrm{Sp}(2n) \times \mathrm{GL}(1)$ . This is a joint work with Yubo Jin.**Esme Rosen, 10:20-10:55***Title:* Certain Hypergeometric Series and  $L$ -values of Modular Forms*Abstract:* Using a framework relating hypergeometric motives to modular forms recently developed by Allen, Long, Grove, and Tu, we define an explicit family of weight 2 Hecke eigenforms with complex multiplication. We use the theory of  ${}_2F_1(1)$  hypergeometric series and Ramanujan's theory of alternative bases to compute the exact central  $L$ -value of these Hecke eigenforms in terms of special beta values. We also briefly will discuss how these results can be generalized to modular forms of weight 3.

**Rajit Gupta, 11:05-11:30**

*Title:* Ramanujan and Koshliakov meets Abel and Plana

*Abstract:* The neglected Russian mathematician, N. S. Koshliakov, derived beautiful generalizations of the classical Abel-Plana summation formula through a setting arising from a boundary value problem in heat conduction. When we let the parameter  $p$  in this setting tend to infinity, his formulas reduce to the classical Abel-Plana summation formula. One particular example provides a vast new generalization of the classical transformation formula for Eisenstein series, which we generalize in Koshliakov's setting.

This is a joint work with Professor Atul Dixit, Professor Bruce C. Berndt, and Professor Alexandru Zaharescu.

**Imin Chen, 11:30-12:30**

*Title:* Generalized Fermat Equations

*Abstract:* The proof of Fermat's Last Theorem pioneered a new approach to resolving families of ternary Diophantine equations using modularity of residual Galois representations attached to Frey curves. In the case of differing exponents, Darmon gave a framework for resolving generalized Fermat equations in one varying exponent using Frey varieties. In this talk, I will survey recent progress on Darmon's program as well as the challenges that remain in the study of generalized Fermat equations.

**Renee Bell, 1:30-2:30**

*Title:* Monodromy of Tame Ramified Covers of Curves

*Abstract:* Galois theory describes the rich connection between field theory and group theory. Similarly, the fundamental group in topology connects group theory to the study of topological spaces. In this talk, we formalize this analogy with the étale fundamental group  $\pi_1(X)$ . Over fields of characteristic 0  $\pi_1(X)$  closely resembles its topological analogue, but in characteristic  $p$ , dramatic differences and new phenomena have inspired many conjectures, including analogues of the inverse Galois problem. Let  $k$  be an algebraically closed field of characteristic  $p$  and let  $X$  be the projective line over  $k$  with three points removed. In joint work with Booher, Chen, and Liu, we show that for each prime  $p \geq 5$ , there are families of tamely ramified covers with monodromy the symmetric group  $S_n$  or alternating group  $A_n$  for infinitely many  $n$ , producing these covers from moduli spaces of elliptic curves.