Moonshine for Finite Groups

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Weak moonshine for a finite group G is the phenomenon where an infinite dimensional graded G-module

$$V_G = \bigoplus_{n \gg -\infty} V_G(n)$$

has the property that its trace functions, known as McKay-Thompson series, are modular functions. Recent work of Dehority, Gonzalez, Vafa, and Van Peski established that weak moonshine holds for every finite group. Since weak moonshine only relies on character tables, which are not isomorphism class invariants, non-isomorphic groups can have the same McKay-Thompson series. We address this problem by extending weak moonshine to arbitrary width $s \in \mathbb{Z}^+$. Namely, for each $1 \leq r \leq s$ and each irreducible character χ_i, we employ Frobenius' r-character extension $\chi_i^{(r)} : G^{(r)} \to \mathbb{C}$ to define McKay-Thompson series of $V_G^{(r)} := V_G \times \cdots \times V_G$ (r copies) for each r-tuple in $G^{(r)} := G \times \cdots \times G$ (r copies). These series are modular functions. We find that *complete* width 3 weak moonshine always determines a group up to isomorphism. Furthermore, we establish orthogonality relations for the Frobenius r-characters, which dictate the compatibility of the extension of weak moonshine for V_G to width s weak moonshine.