Existence of η-quotients of squarefree levels

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In *The Web of Modularity*, Ono observes that every modular form for $\mathrm{SL}_2(\mathbb{Z})$ can be written as a rational expression in η-quotients, and poses the question of which graded rings of modular forms can be similarly generated by η-quotients. Towards answering this question, we give necessary and sufficient conditions for the existence of η-quotients in modular form spaces of weight k and level N. Using these, we exhibit an infinite family of levels N such that the ring $M(\Gamma_0(N))$ is not generated by η-quotients.