Print Your Name Here: ____________________

- **Show all work** in the space provided. We can give credit *only* for what you write! *Indicate clearly if you continue on the back side.*
- **No** books or notes (paper or electronic) or communication devices (smart/cell phones, internet-connected devices such as laptops, tablets, or I-watches) are allowed. A scientific calculator (*not capable* of graphing or symbolic calculations) is allowed—but it is not needed. If you use a calculator, you *must still write out all operations performed* on the calculator. ***Do not*** replace precise answers such as $\sqrt{2}$, $\frac{1}{3}$, or π with decimal approximations. *Keep your eyes on your own paper!*
- The maximum score possible is 100 points.

1. (20 points) Find $\iint_D e^{-x^2}\, dA$ where $D = \{(x, y) \mid 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant x\}$. (Hint: It matters which order of iteration you select.)

2. (20 points) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x\, dx\, dy$ by converting to polar coordinates and integrating.

3. (20 points) *Set up* $\iiint\limits_R x\,dV$ where R is the *first octant* region below the plane $z = 3 - x - 3y$.
Show the *integrand*, the *order* of iteration, and the correct *lower and upper limits* of integration, but, to save time, **do not evaluate the triple integral**.

4. (20 points) Find the volume of the region R that lies *inside* the cone $\phi = \frac{\pi}{6}$ but *below* the sphere $\rho = 1$.

5. (20 points) Let R be the region inside the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$. Use the steps below to find the area of R by integrating $\displaystyle\iint_R 1\,dA$ by *changing variables* as follows: $x = 2u$ and $y = 3v$.

a. Find the *Jacobian* $\dfrac{\partial(x, y)}{\partial(u, v)}$.

b. Find the domain in the uv-plane that gets mapped onto R.

c. Use the information from the first two parts to find the value of $\displaystyle\iint_R 1\,dA$. (You may use your geometrical knowledge of a circle.)

Solutions

1. $\displaystyle\iint_D e^{-x^2}\,dA = \int_0^1\int_0^x e^{-x^2}\,dy\,dx = \int_0^1 xe^{-x^2}\,dx = \frac{e-1}{2e}$, where the last integration requires integration by **substitution**, *not* wild guesses. (Compare with 15.2/13.)

2. (Compare with 15.3/40) $\displaystyle\int_0^1\int_0^{\sqrt{1-y^2}} x\,dx\,dy = \int_0^{\pi/2}\int_0^1 r(\cos\theta)\,rdrd\theta = \frac{1}{3}\int_0^{\pi/2}\cos\theta\,d\theta = \frac{1}{3}$. *Answers need to make sense.* The integrand is strictly positive on the given domain, which has strictly positive area. Thus the integral must be strictly positive. It is very important to recognize that the domain is the part of the unit disc centered at the origin that lies in the *first* quadrant.

3. (Compare with 15.6/17.) $\displaystyle\iiint_R x\,dV = \int_0^1\int_0^{3-3y}\int_0^{3-x-3y} x\,dz\,dx\,dy$ with this choice of order of iteration. The limits could be different if you chose a different order of iteration.

4. $\displaystyle V = \int_0^{2\pi}\int_0^{\frac{\pi}{6}}\int_0^1 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{\pi}{3}(2-\sqrt{3})$. (Compare with 15.8/29.)

5. First, the *Jacobian* $\displaystyle\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$. Then $\displaystyle\iint_R 1\,dA = \iint_{u^2+v^2\leqslant 1} 6\,dudv = 6\pi(1)^2 = 6\pi$. (Compare with 15.9/19.)

Class Statistics

% Grade	Test#1	Test#2	Test#3	Final Exam	Final Grade
90-100 (A)	13	16			
80-89 (B)	6	7			
70-79 (C)	6	4			
60-69 (D)	5	1			
0-59 (F)	0	2			
This Test Avg	84.8%	86.0%	%	%	%
HW Avg	87.5%	88.1%	%	%	%
HW/TST Corr	0.61	0.77			

The Correlation Coefficient is the cosine of the angle between two data vectors in $\mathbb{R}^{31}$–one dimension for each student enrolled. Thus this coefficient is between 1 and -1, with coefficients above 0.6 being considered strongly positive. The correlation coefficient shown indicates that the test grades in the course have a strongly positive correlation with performance on the homework.