Problem Solving Seminar - Fall 2012
Sep. 24

1. Evaluate the following antiderivatives.
   (a) $\int x3^x dx$
   (b) $\int_0^1 te^{3t^2} dt$
   (c) $\int \frac{1}{t^2+2t+3} dt$
2. Suppose that n is a positive integer and $x > 0$. Prove that
$$\frac{x^n}{(x+1)^{n+1}} \le \frac{n^n}{(n+1)^{n+1}}.$$
3. Let f be a continuous function on the entire real line. Prove that if f takes on no value more than twice, it must take on some value exactly once.
4. (a) Show that there are always two antipodal (i.e., directly opposite) points on the earth's equator that are exactly the same temperature.
   (b) A cross-country runner runs a five-mile course in 30 minutes. Prove that somewhere along the course the runner ran a mile in exactly 6 minutes. Is it also true that at some point he ran three miles in exactly 18 minutes?
5. [**1987 B1**] Evaluate
$$\int_2^4 \frac{(\ln(9-x))^{1/2}}{(\ln(9-x))^{1/2} + (\ln(x+3))^{1/2}} dx.$$
6. [**2011 B3**] Let f and g be (real-valued) functions defined on an open interval containing 0, with g nonzero and continuous at 0. If fg and f/g are differentiable at 0, must f be differentiable at 0?
7. [**1999 A5**] Prove that there exists a constant C such that if $p(x)$ is a polynomial of degree 1999, then
$$|p(0)| \le C \cdot \int_{-1}^{1} |p(x)| \, dx.$$