Math 6302: Mathematical Foundations of the Common Core
LaMSTI/CART MNS Degree Program, Summer 2011

Instructors

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Course web site: https://www.math.lsu.edu/~madden/M6302summer2011/index.html

This course is offered in the first summer of the LaMSTI and CART MNS Degree programs for three (3) semester hours of credit under the internal designation *Mathematical Foundations of the Common Core*.

Catalog description

6302 Implementing the NCTM Standards II (3) *May be taken for a max. of 9 sem. hrs. of credit when topics vary. Enrollment is restricted to participants in the teacher-training and grant-supported programs.* Topics for mathematics teachers (6-8) to be selected from those in the Principles and Standards of School Mathematics of the National Council of Teachers of Mathematics.

Comment. The NCTM Standards are no longer as important in setting the agenda for mathematics education as they were when this course description was composed. A petition to revise the course description is being filed, which will remove the reference to NCTM and refer instead to "nationally recognized standards.'' We will use the Common Core State Standards as a basic reference for all work in this course.

Implementation for MNS

This **3-credit-hour** course focuses on the algebra and geometry presented in middle-school and high-school (including dual-credit courses), examined from mathematical, curricular and pedagogical perspectives. Topics include: functions (linear, quadratic, polynomial and exponential), coordinates and analytic geometry, Euclidean geometry, and modeling.

- **Format:** The course is delivered over a period of six weeks in the summer. Students spend 3 to 4 hours per day for a total of at least 26 days. The time is allocated as follows:
    - 1/3 Lecture & activity-based learning,
    - 2/3 Seminar (extended investigations of mathematical topics).
- **Integration with Algebra Lab.** Students in M6300 will enroll simultaneously in a section of M4999, and their assignment in M4999 will include 1.5 hours per day devoted to studying the LSU College Algebra Course, which is delivered on the MyMathLab platform. Assignments in this course will be coordinated with the work in M6302.
- **Grading.** Grading is based on 10 to 15 written assignments, given periodically during the summer. Standards for each assignment will be discussed at the time it is given. Students are expected to work on assignments until the standards are met.

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Core Topics

The following is a list of topics that are prominent in the LSU College Algebra course. These form a minimal core that will be visited both in the M4999 course linked to M6302 and in M6302 itself.

1. Real Numbers
2. Order of Operations and Algebraic Expressions
3. Laws of Exponents and Radicals
4. Polynomials, Factoring Polynomials
5. Linear, Quadratic, and Radical Equations
6. Linear Inequalities
7. Absolute Value
8. Rectangular Coordinates, Circles
9. Lines, Parallel and Perpendicular
10. Functions, Graphs of Functions
11. Piecewise Functions
12. Transformations
13. Composition of Functions, One-to-One and Inverse Functions
14. Quadratic Functions
15. Polynomial Functions and Inequalities
16. Rational Functions and Inequalities
17. Exponential Functions/ The Natural Exponential
18. Logarithmic Functions, Properties of Logs

Expanded topics list. M6302 will also treat a selection of topics from the following lists.

Algebra topics

1. Arithmetic and Algebra
   1.1. Number systems. *Natural numbers, integers, rational numbers. The number line.*
   1.2. Decimal notation. *Algorithms for addition and multiplication. Repeating decimals. Approximation and real numbers.*
   1.3. The rules of arithmetic. *Rings and fields.*
   1.4. Advanced topic: *Euclidean algorithm and continued fractions*
2. Expressions and Equations.
   2.1. Arithmetic expressions. *Structure. Transformations.*
   2.2. Variables and expressions with variables.
   2.3. Equations and solving equations.
   2.4. Word problems.
   2.5. Advanced topic: Descartes' *Geometry*.
3. Coordinates and Graphs
   3.1. Coordinate systems on a line. Solving problems with coordinates. Coordinate transforms.
   3.2. Coordinate systems in the plane. Graphing equations.
   3.3. Lines in the plane.
   3.4. Linear maps from the plane to the plane; matrices.
4. Logic.
   4.1. Connectives ("and", "or" and "not") and logically complex propositions.
   4.2. Existential quantifiers ("for all" and "there exists").
   4.3. Sets defined by logical formulae. Boolean algebra. Products and projections.
   4.4. Advanced topic: Semi-linear and semi-algebraic sets.
5. Functions
   5.1. Definitions and examples. Functions between sets.
   5.2. Linear functions and groups of linear functions.
   5.3. Composition and inversion
   5.4. Exponent and Logarithm.
   5.5. Sequences and recursion.

6. Polynomials and rational functions
   6.1. Computing with polynomials and rational functions.
   6.2. Quadratics.
   6.3. Factoring and the Remainder Theorem.
   6.4. Polynomials of several variables.
   6.5. Conic sections.
   6.6. Advanced topic: Algebraic curves.
7. Word problems and modeling.

Geometry Topics

1. Euclid's *Elements*
   1.1. Informal deductive rigor: definitions, postulates and propositions
   1.2. Book I: Triangles and congruence
   1.3. Book I: The Parallel Postulate and its consequences
   1.4. Book I: Area by dissection
   1.5. Book V: Measurement, ratio and proportion
   1.6. Book VI: Similarity
2. Descartes' *Geometry*
   2.1. Introducing a unit. Multiplying and finding roots by geometry
   2.2. Dynamic figures analyzed using similarity
   2.3. Coordinates
3. Transformations
   3.1. Isometries and similarity transforms: classification
   3.2. Transformation groups
   3.3. Advanced topic: Inversion (in a circle) and hyperbolic geometry

Research Base

Educational research[1, 2, 3] demonstrates that mathematics teachers need:

- a deep understanding of the structure, content and goals of the curriculum,
- a large repertoire of fully analyzed mathematical examples that may be incorporated in lessons and tests, and
- ability to conceptualize and assess the mathematical knowledge of others and select appropriate actions in response.

This course is designed to develop these competencies.

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[1] Ball, D. L., Lubenski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433-456). Washington, DC: American Educational Research Association.

[2] Baumert, J. Kunter, M., Blum, W., Brunner, M, Voss, T., Jordan, A., Klusman, U., Krauss, S., Neubrand, M., & Tsai, Y. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal* 47, 133-180.

[3] Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Hillsdale, NJ: Erlbaum.

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Main Primary Sources

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Descartes, R. *The Geometry of Rene Descartes.* Dover, 1954.

Newton, I. *The Mathematical Papers of Isaac Newton*, Volume I, edited by D.T.Whiteside. Cambridge Univ. Press, 1967.

Other Important Primary Sources

Archimedes. *The Works of Archimedes,* Thomas L. Heath, translator. Dover 2002.

Apollonius. *Conic Sections,* Catesby Taliaferro, translator. Green Lion Press, 1998.

Fibonacci. *Fibonacci's Liber Abaci*, Lawrence Sigler, translator. Springer 2002.

Kepler, J. *Selections from Kepler's Astronomia Nova.* Green Lion Press, 2004.

Newton, I. *The Principia: Mathematical Principles of Natural Philosophy,* I.B.Cohen, trans.. Univ. California Press, 1999.

Secondary Sources

Bashmakova, I. G. & Smirnova, G. S. *The Beginnings and Evolution of Algebra (Dolciani Mathematical Expositions)*. MAA, 2000.

Bashmakova, I. G. *Diophantus and Diophantine Equations (Dolciani Mathematical Expositions)*. MAA, 1998.

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Klein, F., *Elementary mathematics from an advanced standpoint: Geometry*. Dover, 2004.

Lebesgue, H. "Measure of magnitudes," in: *Measure and the Integral by Henri Lebesgue*, Kenneth O. May, editor, Holden-Day, 1966.

Meng, S.K. *New Elementary Mathematics, Syllabus D: books 1, 2, 3A, 3B.* Singapore: Panpac, 2006.

Moise, E. *Elementary geometry from an advanced standpoint, second edition.* Addison-Wesley, 1974.

Tarski, A., *Introduction to logic and the methodology of the deductive sciences.* Dover, 1995.

High School Reference Texts

We will use the four CME Project texts: *Algebra I, Geometry, Algebra II* and *Precalculus.* The CME Project is a four-year, NSF-funded, comprehensive high school mathematics program that is problem-based, student centered, and organized around the traditional high-school course sequence. The series was developed by the Center for Mathematics Education at Education Development Center, Inc. (EDC) in Newton, Massachusetts, and is published by Pearson Education, Inc.

See: http://cmeproject.edc.org/