M2057 Test 1. September 22, 2003 **Name:**

Instructions: There are 8 problems. The maximum score is 105. Except in problems 1, 2.a, 2.b, 4.a, you must show work and/or provide brief explanations.

1. Let $f(x,y) = x\,y$. Write the equations for the level curve that passes through $(2,1)$ and sketch it. Do the same for the level curve through $(0,3)$. (10 pts)

2. Let $g(x,y) = x\,\cos(x\,y)$. Find the following partial derivatives (4 pts each):

   a. $h_x =$

   b. $h_y =$

   c. $h_{xx} =$

   d. $h_{xy} =$

   e. $h_{yy} =$

3. Let $x = s\,t$ and $y = s^2 + t$ and $z = f(x,y)$. Use the chain rule to express $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ in terms of s, t, $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. (10 pts)

4. Suppose $f(x, y, z) = x^2\, y^3\, z^5$ and u is the unit vector $(\frac{1}{2}, 0, \frac{\sqrt{3}}{2})$. Find the following: (5 pts each)

a. $\nabla f =$

b. $\frac{\partial f}{\partial u} =$

c. The value at $(1, 0, 1)$ of the directional derivative of f in the direction of the vector $(1, 2, 2) =$

5. At the point $(2, \pi)$, in what direction does the function $f(x, y) = x + x\,\sin y$ increase most rapidly, and what is the rate of increase? (10 pts)

6. Find the three critical points of $f(x, y) = x^4 + 4xy + 2y^2$. Use the second derivative test to determine which are relative maxima, relative minima and which are neither. (20 pts)

Test continues...

7. Find the equation of the plane tangent to the surface $32 = x^2 + y^3 - z^4$ at $(5, 2, 1)$. (10 pts)

8. Use the method of Lagrange to find the the points where $f(x, y) = 8x + 6y$, subject to the constraint $g(x, y) = 2x^2 + 3y^2 = 11$, has maximum and minimum values and find those values. (10 pts)

The End.