De Broglie geometry on Zeeman manifolds: A new non-perturbative approach to the infinities of QED (Review of the scalar theory)

Zoltán Imre Szabó *

Lehman College and Graduate Center of the City University of New York†

In de Broglie geometry the Hilbert space, where the Hamilton operator acts, is decomposed into invariant subspaces, alias zones, which are separately investigated both from mathematical and physical points of views. The points are non-existing objects on a zone. They rather appear as wave packets (point spreads) there. This implies that also the particles living in a zone are extended ones. Rigorous mathematics shows that quantities appearing as infinities on the total Hilbert space are finite ones on the zonal setting. It is well known that the appearance of infinities in the quantum theory is ultimately due to contradictory concepts such as the point mass and/or point charge. De Broglie geometry ostracizes these infinities by throwing out the point concept and compelling the particles to spread out all over the whole space.

Since these zonal particles can locally interact with the magnetic field present in the AB-solenoid, this model gives a relativistically satisfactory explanation for the Aharanov Bohm effect. The zones are constructed with particular vector potentials. General vector potentials do not define zonal decomposition, thus the zonal structure is not a gauge-invariant concept. The zones attribute physical meaning to the potential by which they are constructed.

PACS numbers: 11.15.Tk, 12.20.-m

I. INTRODUCTION.

This paper consists of two parts. In the first one the Zeeman-Hamilton operator of free charged particles orbiting in a constant magnetic field is established as the Laplacian on a Riemannian, called, Zeeman manifold. The 2D version of this Hamiltonian, introduced by Landau (1928), appears as the Laplacian on a 3D time-periodic Heisenberg group endowed with the natural left-invariant metric.The most general version of Zeeman manifolds are defined, in this paper, on center periodic 2-step nilpotent Lie groups. This paper explores the physical contents offered by this most natural mathematical model. No other objects will be involved into investigations.

The constant magnetic field defined by the invariant Riemannian metric determines a unique inertia system, along with self-time, for each particle, in which the field has vanishing electric field E. This constant magnetic field, so to speak, furnishes this model with relativistic features, relating it to Dirac's relativistic multi-time model. It is remarkable that this model leads to probabilistic quantum theory working with positive probabilities defined just on the space. Dirac's relativistic electron theory, which is not the same as his multi-time theory, establishes such positive probabilities on the Minkowski space time. This idea was sharply criticized by Pauli, according to whom such probabilistic theory makes sense only on the space. The problem is that no clear descending from the space time level to the space is offered by the Minkowski geometry. It can be defined, however, in our model. The transmission from the whole group (space multitime) to the space is called Pauli transmission. This concept is nothing to do with the Riemannian submersion theory.

The second theme of the paper involves and further develops the Fock-Bargmann theory on Zeeman manifolds. The FB-representation of the complex Heisenberg group is originally considered only on the Fock space, generated by the holomorphic polynomials in the total space of complex valued functions. No other invariant subspaces of this reducible representation have been investigated in the literature so far. This paper explores all irreducible subspaces, called Zeeman zones, of the FB-representation. This zonal spectral analysis includes the explicit description of various zonal objects such as the projection kernels, the zonal spectra, and the zonal Wiener- resp. Schrödinger-flows. The most surprising result is that both zonal flows are of the trace class, defining the zonal partition and zeta functions in the standard way, by using no renormalization. Even the zonal Feynman measures on the path-space are well defined. In other words, quantities appearing as infinities on the global level are well defined finite ones on the zonal setting.

II. ZEEMAN MANIFOLDS

A. Zeeman-Hamilton operators.

The classical Zeeman operator of a charged particle is

$$H_Z = -\frac{\hbar^2}{2\mu}\Delta - \frac{\hbar eB}{2\mu c\mathbf{i}}D_z\bullet + \frac{e^2B^2}{8\mu c^2}(x^2+y^2) + eV, \qquad (1)$$

where V is the Coulomb potential originated from the nucleus (for free particles $V=0$ holds). This operator is usually considered on the 3-space. The free particle operator restricted onto the (x,y)-plane (i. e., $V=0$ and Δ is the Euclidean Laplacian on $\mathbf{R}^2$) is called Landau Hamiltonian. This paper proceeds with this version of the Zeeman operator. Operator $D_z\bullet = x\partial_y - y\partial_x$ is the so called angular momentum operator. The $D_z\bullet$ commutes with the rest part, $\mathbf{O}$, of the complete operator, thus the spectrum appears on common eigenfunctions.

---

*Research partially supported by NSF grant DMS-0604861.

†Also at Rényi Institute of Mathematics, Budapest, Hungary; Electronic address: zoltan.szabo@lehman.cuny.edu

I. e., the $D_z \bullet$ splits the spectral lines of **O** which phenomena is associated with the Zeeman effect. Actually, the H_Z is the Hamilton operator of an electron orbiting about the origin of the (x,y)-plane in a constant magnetic field $\mathbf{K} = B\partial_z$. The 3D-version can be established by means of the Maxwell equations and the real Heisenberg group representation. To establish the Landau Hamiltonian, one can use the Fock-Bargmann representation of the complex Heisenberg group. The next considerations proceed with the Landau version of the Zeeman operator defined over a complex vector space $\mathbf{C}^{k/2} = \mathbf{R}^k$.

B. Mathematical modeling: Zeeman manifolds.

Interestingly enough, the Landau operator, H_Z, can be identified with the Laplace operators of two step nilpotent Lie groups endowed with the natural left invariant metrics. As far as the author knows, this interpretation is unknown in the literature. A 2-step nilpotent metric Lie group is defined on the product $\mathbf{v} \oplus \mathbf{z}$ of Euclidean spaces, where the components, $\mathbf{v} = \mathbf{R}^k$ and $\mathbf{z} = \mathbf{R}^l$, are called X- and Z-space respectively. The Lie algebra is completely determined by the linear space, $J_{\mathbf{z}}$, of skew endomorphisms acting on the X-space defined by $\langle [X,Y],Z\rangle = \langle J_Z(X),Y\rangle$, where $X,Y \in \mathbf{v}$ and J_Z is the endomorphism associated with $Z \in \mathbf{z}$. The metric, g, is the left invariant extension of the natural Euclidean metric on the Lie algebra. The exponential map identifies the Lie algebra with the group. Thus also the group can be considered such that it is defined on the same vector space. By this identification, a point is represented by (X,Z) also on the group.

Particular 2-step nilpotent Lie groups are the Heisenberg-type groups, introduced by Kaplan [K], defined by endomorphism spaces satisfying the Clifford condition $J_Z^2 = -|Z|^2 id$. These metric groups are attached to Clifford modules, thus the classification of these modules provides classification also for the H-type groups. In this case the X-space decomposes into the product $\mathbf{v} = (\mathbf{R}^{r(l)})^{a+b} = \mathbf{R}^{r(l)a} \times \mathbf{R}^{r(l)b}$. Endomorphisms J_Z are defined by endomorphisms j_Z acting on the smaller space $\mathbf{R}^{r(l)}$. Namely, the J_Z acts on $\mathbf{R}^{r(l)a}$ resp. $\mathbf{R}^{r(l)b}$ as $j_Z \times \cdots \times j_Z$ resp. $-j_Z \times \cdots \times -j_Z$. The H-type groups are denoted by $H_l^{(a,b)}$, indicating the above decomposition.

The Laplacians on H-type groups are of the form

$$\Delta = \Delta_X + (1 + \frac{1}{4}|X|^2)\Delta_Z + \sum_{\alpha=1}^{r} \partial_\alpha D_\alpha \bullet, \qquad (2)$$

where $D_\alpha \bullet$ denotes directional derivatives along the fields $J_\alpha(X) = J_{Z_\alpha}(X)$ and $\{Z_\alpha\}$ is an orthonormal basis on the Z-space. This operator is not the Landau operator yet. It appears, however, on center periodic H-type groups, $\Gamma\backslash H$, defined by factorizing the center of the group with a Z-lattice $\Gamma = \{Z_\gamma\}$. In fact, in this case the L^2 function space is the direct sum of function spaces W_γ spanned by functions of the form $\Psi_\gamma(X,Z) = \psi(X)e^{2\pi\mathbf{i}\langle Z_\gamma,Z\rangle}$. Each W_γ is invariant under the action of the Laplacian, i. e., $\Delta\Psi_\gamma(X,Z) = \Box_\gamma\psi(X)e^{2\pi\mathbf{i}\langle Z_\gamma,Z\rangle}$, where operator $\Box_\gamma$, acting on $L^2(\mathbf{v})$, is of the form

$$\Box_\gamma = \Delta_X + 2\pi\mathbf{i}D_\gamma \bullet - 4\pi^2|Z_\gamma|^2(1 + \frac{1}{4}|X|^2). \qquad (3)$$

Notice that the Landau Hamiltonian can be identified with $H_Z = -(1/2)\Box_\gamma$, defined for the 3D-Heisenberg group, by choosing the constants in the form $\mu = \hbar = 1, \pi|Z_\gamma| = \lambda = eB/2c$. However, operator (3) contains also the surplus constant $4\pi^2|Z_\gamma|^2 = 4\lambda^2$, which does not appear in the Landau Hamiltonian. This constant can be interpreted as the total energy of the field attached to a zonal particle. On a $(k+1)$-dimensional Heisenberg group, defined by a complex structure J acting on the even dimensional Euclidean space $\mathbf{v} = \mathbf{R}^k$, number $k/2$ is interpreted as the number of particles represented by the system. The Zeeman operator appears as Laplacian on general center periodic 2-step nilpotent Lie groups in a bit more complex form. The $k/2$ particles, represented by this model, are orbiting in their own constant magnetic fields. The system can be in crystal states represented by the endomorphisms J_γ. The Hamilton operators belonging to these crystal states are $-\frac{1}{2}\Box_\gamma$.

There is described in the Introduction how this model can be associated with Dirac's famous multi-time theory, which, in order to establish a relativistic quantum theory, attributed self-time to the particles. The unique inertia system and self time is defined for each crystal state associated with Z_γ separately. The line spanned by Z_γ and parametrized by the arc-length t can be interpreted as the self-time axis in the inertia system. By considering this construction on the non-periodic group for each element Z of the center, operator (2) can be interpreted as the Gordon-Klein operator on this relativistic model.

Pauli's criticism of Dirac's relativistic electron equation shows that an adequate relativistic space-time model on the quantum level requires substantial modification of the Minkowski space-time concept. Particularly, he was missing a transmission by which one can descend from the space time level to the space level. Our model solves this problem by a canonical choice of a unique inertia system by means of the constant magnetic field defined by the invariant Riemannian metric. The physical system is described in this inertia system. The above construction clearly demonstrates the Pauli transmission of the Gordon-Klein operator (2), which still contains operations regarding the Z-variables, to the Landau Hamiltonian, (3), acting on functions depending just on the X-variable. The operations regarding the center variables can be eliminated because the Fourier functions defined by the lattice Γ on the center are eigenfunctions of operators Δ_Z and ∂_α. Note that this transmission is established for any invariant subspace W_γ separately by the map $\pi_\gamma : W_\gamma \to \mathscr{H}$; $\pi_\gamma : \Psi_\gamma(X,Z) \to \psi(X)$, where Hilbert space $\mathscr{H}$ consisting functions depending just on the X-variable is introduced in the next paragraph. The probabilistic quantum theory is established by the Landau Hamiltonian, which defines positive probabilities on the X-space.

This paper deals also with the corresponding theory on the non-periodic groups. The Riemannian manifolds introduced so far are prototypes of a general Zeeman manifold concept. This general concept is beyond the scope of this review. The Riemannian manifolds depicted above aroused, originally, in constructing isospectral isospctral manifolds with different local geometry [Sz1]-[Sz4]. The isospectrality deformations introduced in these papers will be used to develop a theory of symmetries on the quantum level.

III. NORMAL DE BROGLIE GEOMETRY

A. Introducing the zones.

The Laplacian $\Box_\lambda$ acts on complex valued functions defined on the X-space. In what follows the investigations are performed on the X-space by considering the Laplacian $\Box$ for a fixed lattice point Z_γ. There is described in Section III E that how these considerations can be "transported" to the whole (X,Z)-space.

The Hilbert space, $\mathscr{H}$, of the L^2-functions is isomorphic to the weighted space defined by the Gauss density $d\eta_\lambda(X) = e^{-\lambda|X|^2}dX$. The latter space is spanned by the complex valued polynomials. Next $\mathscr{H}$ is considered in this form. The natural *complex Heisenberg group representation* on $\mathscr{H}$ is defined by

$$\rho_{\mathbf{c}}(z_i)(\psi) = (-\partial_{\overline{z}_i} + \lambda z_i\cdot)\psi \quad , \quad \rho_{\mathbf{c}}(\overline{z}_i)(\psi) = \partial_{z_i}\psi, \qquad (4)$$

where $\{z_i\}$ is a complex coordinate system on the X-space. This representation is reducible. In fact, it is irreducible on the Fock space generated by the holomorphic polynomials. There it is called Fock-Bargmann representation. Besides the Fock space there are infinitely many other irreducible invariant subspaces. The above representation is called *extended Fock-Bargmann representation*. In the function operator correspondence, this representation associates operator (1) to the Hamilton function of an electron orbiting in constant magnetic field.

The zones are defined in two different ways. First, they can be defined by the invariant subspaces of representation (4). The actual construction uses Gram-Schmidt orthogonalization. On the complex plane $\mathbf{v} = \mathbf{C}$, corresponding to the 2D Landau operator, the $\mathscr{H}$ is the direct sum of subspaces $G^{(a)}$ spanned by functions of the form $\overline{z}^a h$, where h is an arbitrary holomorphic polynomial. Then one gets the zones $\mathscr{H}^{(a)}$, where $a = 0,1,2,\dots$, by the Gram-Schmidt orthogonalization process applied to the function spaces $G^{(a)}$. It is clear that the first zone, $\mathscr{H}^{(0)}$, is the Fock space. The zone index a indicates the maximal number of the antiholomorphic coordinates $\overline{z}$ in the polynomials spanning the zone.

One of the referees of [Sz6] pointed out to me that the polynomials produced by this constructions were considered also by Itô [I] in the context of complex Markov processes. In fact, Itô defines the Hermite polynomials of complex variables for $p,q = 0,1,2,\dots$ by the explicit formula

$$H_{pq}(z,\overline{z}) = \sum_{n=0}^{p\wedge q}(-1)^n \frac{\underline{|p}\underline{|q}}{\underline{|n}\underline{|p-n}\underline{|q-n}} z^{p-n}\overline{z}^{q-n}, p\wedge q = min(p,q). \qquad (5)$$

In the 2D case, they form an orthonormal basis in $\mathscr{H}$ defined for $\lambda = 1$. In this formalism, the zones are spanned by polynomials belonging to fixed values of q, i. e., the q corresponds to the zone index a in our formalism.

The construction with the Gram Schmidt orthogonalization easily extends to general dimensions. Gross zone $\mathscr{H}^{(a)}$ is constructed by means of all polynomials $\overline{z}_1^{(a_1)}\dots\overline{z}_{k/2}^{(a_{k/2})}$ satisfying $a_1 + \dots + a_{k/2} = a$. This gross zone is the direct sum of the subzones $\mathscr{H}^{(a_1\dots a_{k/2})}$ defined for the particular values $a_1,\dots,a_{k/2}$. In the 2D-case all the zones are irreducible under the action of the extended Fock-Bargmann representation. In the higher dimensions, however, the *gross zones* are reducible and the subzones are irreducible. Note that the holomorphic (Fock) zone is always irreducible. For the sake of simplicity, all the formulas below are established on the gross zones.

The second technique explores that the zones are invariant under the action of the Laplacian and defines the very same zones by computing the spectrum and the corresponding eigenfunctions explicitly. According to these computations, the eigenfunctions appear in the form $h^{(p,\upsilon)}(X) = H^{(p,\upsilon)}(X)e^{-\lambda|X|^2/2}$ with the corresponding eigenvalues $-((4p+k)\lambda + 4k\lambda^2)$, where p resp. υ are the holomorphic resp. antiholomorphic degrees of polynomial $H^{(p,\upsilon)}$. Numbers $l = p+\upsilon$ and $m = 2p-l$ are called azimuthal and magnetic quantum numbers (AQN and MQN) respectively. The above function is an eigenfunction also of the magnetic dipole moment operator with eigenvalue m. Then a zone is spanned by eigenfunctions having the same index υ. According to the formula $\upsilon = \frac{1}{2}(l-m)$, the zones are determined by the magnetic quantum number m. Thus a zone exhibits the magnetic state of the particle. Note that eigenvalues are independent of the antiholomorphic index and they depend just on the holomorphic index. As a result, each eigenvalue has infinite multiplicity. On the irreducible zones, however, each multiplicity is $k/2$. Moreover, two irreducible zones are isospectral.

It is noteworthy that the above spectrum computation is not the standard one, where the eigenfunctions are sought in the form $f(|X|)G^{(l)}$, where $G^{(l)}$ is an l^{th}-order homogeneous harmonic polynomial. Indeed, in the 2D case, the homogeneous harmonic polynomials are of the form z^p or $\overline{z}^q$ which shows the differences between the two calculations. In [Sz6] the eigenfunctions are explicitly computed in both ways. In quantum theory the azimuthal quantum number is defined by the order of G. Thus it is different from the above AQN which involves some contribution also from the radial part. However, one defines the very same magnetic quantum numbers in both ways.

It is very intriguing that functions (5) are eigenfunctions of the Landau Hamiltonian. (This statement is not quite obvious and the presentations given in (5) and [Sz6] should be considered as equivalent and not the same ones.) As it is pointed out by the above referee, the motivation for these polynomials in Itô's work is quite different and this connection between the two fields seems to be unknown in the literature. This is probably due to the standard computational techniques applied in spectral theory by which Itô's polynomials are not "visible".

B. Projection kernels and point-spreads.

In the literature only the projection onto the Fock space $\mathcal{H}^{(0)}$ is well known, which turned out to be a convolution operator with the so called Fock-Bargmann kernel $(\lambda/\pi)^{k/2}e^{\lambda(z\cdot\overline{w}-\frac{1}{2}(|z|^2+|w|^2)}$. Our theory, developed in [Sz5, Sz6], explicitly determines the projection also onto a general zone $\mathcal{H}^{(a)}$, regarding of which the projection kernel is

$$\delta^{(a)}_{\lambda z}(w)=(\lambda/\pi)^{k/2}L_a^{((k/2)-1)}(\lambda|z-w|^2)e^{\lambda(z\cdot\overline{w}-\frac{1}{2}(|z|^2+|w|^2)}, \tag{6}$$

where $L_a^{((k/2)-1)}(t)$ is the Laguerre polynomial indicated by the indexes. To have this formula, consider an orthonormal basis $\{\varphi_i^{(a)}\}_{i=1}^{\infty}$ formed by eigenfunctions being in $\mathcal{H}^{(a)}$. The projection kernel can be formally expressed in the form $\delta^{(a)}(z,w)=\sum_i\varphi_i^{(a)}(z)\overline{\varphi_i^{(a)}(w)}$, where z and w represent complex vectors on $\mathbf{C}^{\frac{k}{2}}=\mathbf{R}^k$. Then the formula can be established by means of the explicit eigenfunctions. These kernels can be interpreted as restrictions of the global Dirac delta distribution, $\delta_z(w)=\sum\varphi_i(z)\overline{\varphi}_i(w)$, onto the zones.

These kernels represent one of the most important concepts in this theory. They can be interpreted such that, on a zone, a point particle appears as a spread described by the above wave-kernel. Note that how these kernels, called zonal point-spreads, are derived from the one defined for the holomorphic (Fock) zone. This holomorphic spread, which involves a Gauss function, is just multiplied by the radial Laguerre polynomial corresponding to the zone. This form of the functions describing the point-spreads show the most definite similarity to the de Broglie wave packets. In a rigorous theory, function $\delta^{(a)}_{\lambda Z}\overline{\delta}^{(a)}_{\lambda Z}$ is the density of the point-spread concentrated around Z and $\delta^{(a)}_{\lambda Z}$ is the so called spread-amplitude. On a given zone the point-spreads are the most compressed wave packets, yet they are distributed all over the whole space. This zonal particle theory gives a clear explanation for the Aharanov-Bohm (AB) effect [AB] as well as other phenomenas described in [Sz5].

The AB effect produces relative phase shift between two electron beams enclosing a magnetic flux even if they do not touch the magnetic field. This effect has no explanation in the classical mechanics and it contradicts even the relativistic principle of *all fields must interact only locally*. Yet, this effect was clearly demonstrated by the *Tonomura et al experiments* [T1, T2].

Although the point electrons do not touch the fields, the vector potential involved into the Hamilton operator of the system does reach there. Exploiting this phenomena, Aharanov and Bohm explained the effect by the "significance of electromagnetic potentials in the quantum theory". In classical physics this potential is considered to be a mere mathematical convenience which is completely meaningless from physical point of view. In de Broglie geometry the zonal particles are extended ones which must touch the magnetic field, which is a clear enough explanation for the AB effect. Since the zones are defined by a particular vector potential, this explanation is in accordance with the Aharanov-Bohm idea. Indeed, the vector potential is not just a mathematical convenience any more but it is one of the important physical objects by which the zonal structure is defined.

Despite the clear demonstration that the experiments were performed under the condition of complete confinement of the magnetic field in the magnet, some physicists have questioned the validity of the tests, attributing the phase shift to leakage fields. The electron spread idea developed in this paper can be interpreted such that not the magnetic field but "the electrons are leaking". By the uncertainty principle, it is impossible to design an apparatus to determine the leakage of electrons, that will not at the same time disturb the electrons enough to destroy the interference pattern demonstrating the AB effect. Thus our reasoning for explaining the AB effect is indisputable, both logically and experimentally.

C. Global Wiener- and Schrödinger-flows.

In order to continue with the zonal analysis, first, we describe the global flows, defined on the total Hilbert space $\mathcal{H}$. The global Wiener-flow, $e^{-tH_Z}(t,X,Y)$, appears in the following explicit form:

$$\Big(\frac{\lambda}{2\pi sinh(\lambda t)}\Big)^{k/2}e^{-\lambda(\frac{1}{2}coth(\lambda t)|X-Y|^2+\mathbf{i}\langle X,J(Y)\rangle}. \tag{7}$$

This kernel satisfies the Chapman-Kolmogorov identity and it tends to $\delta(X,Y)$ when $t\to 0_+$. However, it is not of the trace class, thus functions such as the partition function or the zeta function are not defined in the standard way. Also note that by regularization (renormalization) only well defined relative(!) partition and zeta functions are introduced.

The global Schrödinger kernel, $e^{-\mathbf{i}tH_Z}(t,X,Y)$, appears in the following explicit form:

$$\Big(\frac{\lambda_i}{2\pi\mathbf{i}sin(\lambda t)}\Big)^{k/2}e^{\mathbf{i}\lambda\{\frac{1}{2}cot(\lambda t)|X-Y|^2-\langle X,J(Y)\rangle\}}. \tag{8}$$

Since for fixed t and X the function depending on Y is not L^2, the integral required for the Chapman-Kolmogorov identity is not defined for this kernel. Neither is this kernel of the trace class. Nevertheless, it satisfies the above limit property when $t\to 0_+$.

It is well known that rigorously defined measure on the path-spaces can be introduced only with the Wiener kernel e^{-tH}. Note that the heat kernel involves a Gauss density which makes this constructions possible. Whereas, the Schrödinger kernel does not involve such term. This is why no well defined constructions can be carried out with this kernel. These difficulties disappear, however, by considering these constructions on the zones separately.

D. Zonal Wiener- and Schrödinger-flows.

The zones are invariant with respect to the action of the Hamilton (Laplace) operator, thus the zonal flows are well defined on each zone. The zonal Wiener-kernels are of the trace class, which can be described by the following explicit formulas.

$$e^{-tH_Z^{(0)}}=\Big(\frac{\lambda e^{-\lambda t}}{\pi}\Big)^{\frac{k}{2}}e^{\lambda(-\frac{1}{2}(|X|^2+|Y|^2)+e^{-2\lambda t}\langle X,Y+\mathbf{i}J(Y)\rangle)}, \quad (9)$$

$$e^{-tH_Z^{(a)}}=\mathscr{L}_a^{(\frac{k}{2}-1)}(t,X,Y))e^{-tH_Z^{(0)}}(t,X,Y), \quad (10)$$

where $\mathscr{L}_a^{(\frac{k}{2}-1)}$ can be explicitly computed in terms of the regarding Laguerre polynomial and e^{-2t}. Furthermore, for the zonal partition function, $Tre^{-tH_Z^{(a)}}$, we have

$$\mathscr{Z}_1^{(a)}(t)=\binom{a+(k/2)-1}{a}e^{-\frac{k\lambda t}{2}}/(1-e^{-2\lambda t})^{\frac{k}{2}}. \quad (11)$$

Also the zonal Schrödinger kernels are of the trace class which, together with their partition functions, can be described by the following explicit formulas.

$$e^{-t\mathbf{i}H_Z^{(0)}}=\Big(\frac{\lambda e^{-\lambda t\mathbf{i}}}{\pi}\Big)^{\frac{k}{2}}e^{\lambda_i(-\frac{1}{2}(|X|^2+|Y|^2)+e^{-2\lambda_i t\mathbf{i}}\langle X,Y+\mathbf{i}J(Y)\rangle)}, \quad (12)$$

$$e^{-t\mathbf{i}H_Z^{(a)}}=\mathscr{L}_{\mathbf{i}a}^{(\frac{k}{2}-1)}(t,X,Y))e^{-t\mathbf{i}H_Z^{(0)}}(t,X,Y), \quad (13)$$

$$\mathscr{Z}_{\mathbf{i}}^{(a)}(t)=\binom{a+(k/2)-1}{a}e^{-\frac{k\lambda t\mathbf{i}}{2}}/(1-e^{-2\lambda t\mathbf{i}})^{\frac{k}{2}} \quad (14)$$

The zonal Schrödinger-kernels are zonal fundamental solutions of the Schrödinger equation, satisfying the Chapman-Kolmogorov identity and tending to $\delta^{(a)}$ when $t\to 0_+$.

On the zones the W- and the Sch-kernels are not just of the trace class. They both define, rigorously, complex zonal measures, the zonal Wiener measure $dw_{1xy}^{T(a)}(\omega)$ and the zonal Feynman measure $dw_{\mathbf{i}xy}^{T(a)}(\omega)$, on the space of continuous curves $\omega:[0,T]\to\mathbf{R}^k$ connecting two points x and y. The existence of zonal W-measure is not surprising, since this measure exists even for the global setting. However, the trace class property is a new feature, indeed. In case of the zonal Feynman measure both the trace class property and the existence of rigorously defined zonal Feynman measures are new features indeed. Note that also the zonal Sch-kernels involve a Gauss kernel which makes these constructions well defined.

E. The non-periodic zones defined by Fourier-averaging.

On center periodic 2-step nilpotent Lie groups the invariant subspaces W_γ, defined for a lattice point Z_γ by functions of the form $\Psi_\gamma(X,Z)=\psi(X)e^{2\pi\mathbf{i}\langle Z_\gamma,Z\rangle}$, is identified, by the map $\Psi_\gamma\to\psi$, with function space $\mathscr{H}$ consisting of functions depending just on the X-variable. Although the zonal decomposition is established on $\mathscr{H}$, it depends on γ and it lives, actually, on W_γ. By considering this zonal decomposition on each W_γ, it lives on $L^2(\Gamma\backslash H)$.

Such simple reduction to the X-space is not possible on non-periodic groups. Unlike in the periodic case, where the zonal functions involve just one function, $e^{2\pi\mathbf{i}\langle Z_\gamma,Z\rangle}$, which depend on the Z-variable, the zonal functions in a zone on the non-periodic manifolds involve all the functions which depend on the Z-variable. Next we describe this construction just on the H-type groups.

In the first step, for any unit vector V_u of the Z-space, consider a complex orthonormal basis $\{Q_{V_u1},\dots,Q_{V_uk/2}\}$ on the complex X-space defined by the complex structure J_{V_u} which defines the complex coordinate system $\{z_{V_u1},\dots,z_{V_uk/2}\}$ on the X-space. This basis field must be smooth on an everywhere dense open subset of the unit Z-sphere such that it is the complement of a set of 0 measure. For given values $a_1,\dots,a_{k/2}$ satisfying $a_1+\cdots+a_{k/2}=a$ consider the zone, $\mathscr{H}_{V_u}^{(a_1\dots a_{k/2})}$, defined by $\overline{z}_{V_u1}^{(a_1)}\dots\overline{z}_{V_uk/2}^{(a_{k/2})}$ by the Gram Schmidt orthogonalization. Then the straight zone, $\mathscr{S}^{(a_1\dots a_{k/2})}$, is spanned by functions of the form $\int_{\mathbf{R}^l}e^{\mathbf{i}\langle Z,V\rangle}\phi(V)h_{V_u}^{(a_1\dots a_{k/2})}dV$, where $\phi(V)$ is an L^2-function defined on the Z-space $\mathbf{R}^l$ and $h_{V_u}^{(a_1\dots a_{k/2})}$ is eigenfunction (Itô-function) from the corresponding zone $\mathscr{H}_{V_u}^{(a_1\dots a_{k/2})}$.

It can be shown that the L^2 Hilbert space on the whole group H is the direct sum of the straight zones $\mathscr{S}^{(a_1\dots a_{k/2})}$. The spectral investigations on these zones are much more complicated then on the zones defined for center periodic groups. For indicating the difficulties we mention that the eigenfunctions of the Gordon-Klein Laplacian Δ are of the form $\oint_{S_{R_Z}}e^{\mathbf{i}\langle Z,V\rangle}\phi(V)h_{V_u}^{(a_1\dots a_{k/2})}dV$, where S_{R_Z} is a sphere of radius R_Z around the origin of the Z-space and $\phi(V)$ is an L^2-function defined on this sphere. This formula shows that the spectrum of the operator is continuous and each eigenvalue has infinite multiplicities. The spectral analysis with such a complicated spectrum will be developed elsewhere.

F. Infinities in Quantum Electrodynamics.

The problem of infinities (divergent integrals), which is present in calculations since the early days of quantum field theory (cf. Heisenberg-Pauli (1929-30)) or elementary particle physics (cf. Oppenheimer (1930) and Waller (1930) in electron theory), is treated by *renormalization* in the current theories. This perturbative tool provides the desired finite quantities by differences of infinities. This problem is the legacy of controversial concepts such as *point mass* and *point charge* of classical electron theory, which provided the first warning that a point electron will have infinite electromagnetic self-mass: the mass $e^2/6\pi ac^2$ for a surface distribution of charge with radius a blows up for $a\to 0$.

The infinities, related to the divergence of the summations over all possible distributions of energy/momentum of the virtual particles, mostly appear in the form of infinite traces of kernels such as the W-kernel e^{-tH} or the Sch-kernel $e^{-tH\mathbf{i}}$. The new non-perturbative approach, described in this paper, to the problem of infinities is called de Broglie geometry. This name readily suggests that a point, x, is a non-existing object on a zone. It rather appears there as a point spread defined by projecting the Dirac delta, δ_x, onto the zone. I. e., a point becomes a wave packet on a zone whose explicit form exhibits its very close kinship to the de Broglie waves. In a sense, de Broglie geometry ostracizes the infinities by exchanging the points for wave packets. This paper reviews the first part of a mathematically complete theory [Sz6]. This theory extends to Anomalous de Broglie Geometry, established for the Pauli-Dirac operator, as well as to Bounded Particle Theory, where the Coulomb potential is eliminated and replaced by potentials defined by the curvature on the X-space.

[AB] Y. Aharanov and D. Bohm: Significance of electromagnetic potentials in the quantum theory. *Phys. Rev.*, 115:485–491, 1959.

[AC] Y. Aharanov and A. Casher: Ground state of spin-$1/2$ charged particle in a two-dimensional magnetic field. *Phys. Rev. A*, 19:2461-2462, 1979.

[B] V. Bargmann: On a Hilbert space of analytic functions and an associated integral transform. Part I. *Comm. Pure Appl. Math.*, 14:187-214, 1961

[Foc] V. Fock: Konfigurationsraum und zweite Quantelung. *Zeit. für Fys.* 75:622-647, 1932.

[I] K. Itô: Complex multiple Wiener integral. *Jap. J. Math.* 22:63–86 (1953).

[K] A. Kaplan: Riemannian nilmanifolds attached to Clifford modules. *Geom. Dedicata*, 11:127–136, 1981.

[LL] L. D. Landau, E. M. Lifshitz: Quantum Mechanics. Pergamon Press LTD, 1958.

[Sh] I. Shigekawa: It-Wiener expansions of holomorphic functions on the complex Wiener space. *Stochastic analysis*, 459–473, Academic Press, Boston, MA, 1991.

[Su] H. Sugita: Holomorphic Wiener function. New trends in stochastic analysis (Charingworth, 1994), 399–415, World Sci. Publishing, River Edge, NJ, 1997.

[Sz1] Z. I. Szabó: Locally non-isometric yet super isospectral spaces. *Geom. funct. anal. (GAFA)*, 9:185–214, 1999.

[Sz2] Z. I. Szabó: Isospectral pairs of metrics on balls, spheres, and other manifolds with different local geometries. *Ann. of Math.*, 154:437–475, 2001.

[Sz3] Z. I. Szabó: A cornucopia of isospectral pairs of metrics on spheres with different local geometries. *Ann. of Math.*, 161:343–395, 2005.

[Sz4] Z. I. Szabó: Reconstructing the intertwining operator and new striking examples added to "Isospectral pairs of metrics on balls and spheres with different local geometries". *DG/0510202 (submitted)*

[Sz5] Z. I. Szabó: Theory of zones on Zeeman manifolds: A new approach to the infinities of QED. *DG/0510660 (submitted)*

[Sz6] Z. I. Szabó: Normal zones on Zeeman manifolds with trace class heat and Feynman kernels and well defined zonal Feynman integrals. *math.SP/0602445 (submitted)*

[Sz7] Z. I. Szabó: Pauli-Dirac operators and anomalous zones on Zeeman manifolds. submitted.

[T1] A. Tonomura, et al: Observation of Aharanov-Bohm effect by electron holography. *Phys. Rev. Lett.*, 48:1443–1446, 1982.

[T2] A. Tonomura, et al: Evidence for Aharanov-Bohm effect with magnetic field completely shielded from electron wave. *Phys. Rev. Lett.*, 56:792-795, 1986.