Dimension Functions of Rationally Dilated Wavelets

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Abstract

In their paper *A characterization of dimension functions of wavelets*, Bownik, Rzeszotnik and Speegle derive a formula for the dimension function of any wavelet in $L^2(\mathbb{R}^N)$ whose associated dilation has integer entries. Furthermore, they give four conditions which characterize all dimension functions of such wavelets. We wish to extend these results to the case of rationally dilated wavelets in $L^2(\mathbb{R}^N)$. Presently, we do so for the specific case where the space of negative dilates is shift invariant. We accomplish this by showing that the formula derived by Bownik, Rzeszotnik and Speegle still holds for this larger class of wavelets while also showing the characterizing conditions (specifically the consistency condition) must be generalized slightly to account for the fact that $A^T\mathbb{Z}^N$ may not be contained in $\mathbb{Z}^N$.