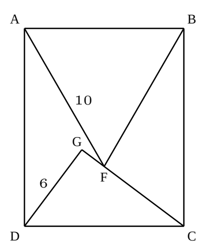
- No calculators are allowed.
- Pictures are only sketches and are not necessarily drawn to scale or proportion.
- You have one hour and fifteen minutes to complete the entire team session.

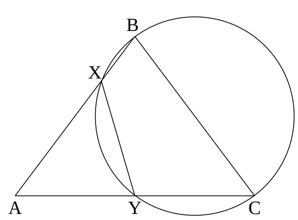
These 10 problems require exact numerical or algebraic answers. Exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers.

The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage in case of a tie.*

1. Suppose $ABCD$ is a rectangle, $\triangle ABF$ is equilateral, and $\triangle DGC$ is a right triangle. Suppose the vertex of the equilateral triangle is on a side of the right triangle at point F. See the diagram below. If $|DG| = 6$ and $|AF| = 10$ what is $|AD|$?



2. In triangle ABC, $|AB| = |BC| = r$ and $|AC| = s$. The circle with diameter BC intersects AB at X and AC at Y. Determine the length of XY. Your answer will depend on the parameters given.

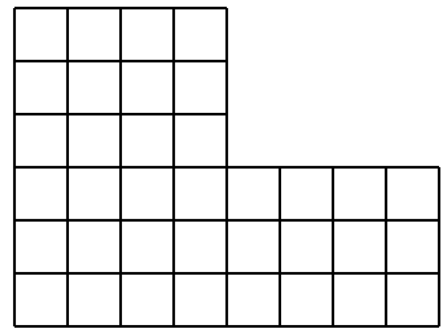


3. Define
$$a_k = \left\lfloor \frac{10^k}{7} \right\rfloor - 10 \left\lfloor \frac{10^{k-1}}{7} \right\rfloor,$$
where
$$\lfloor x \rfloor = n \text{ if } n \le x < n+1.$$
Find a_{2019}.

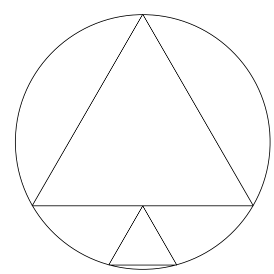
4. How many numbers between $7,000,000$ and $8,000,000$ are there for which the millions digits equals the sum of the other six digits.

5. A certain positive integer is 12 times the sum of its digits (when written in base 10). What is the number?

6. How many **squares** can be formed from the grid lines in the following figure given that all adjacent grid lines are equally spaced.



7. An equilateral triangle is inscribed in a circle of radius r. Another smaller equilateral triangle is inscribed in the region bounded by the base of the the equilateral triangle and the lower part of the circle in such a way that the heights of each triangle are on the same line. See the picture below. What is the ratio of the area of the larger triangle to the area of the smaller triangle?



8. If $[a_1, a_2, a_3, \ldots]$ is a sequence of positive integers we can construct the following so called **continued fraction**
$$\cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \ldots}}}}$$
Compute $[2, 4, 2, 4, 2, 4 \ldots]$

9. Suppose a, b, c are integers and $0 < a < b < c$. In addition, suppose the polynomial $x(x-a)(x-b) - 2019$ is divisible by $x - c$. Find $a + b + c$.

10. A certain number M has a 3 as its unit digit when written in the decimal expansion. A new number N is formed from M by moving the unit digit to the left most position. For example, if $M = 213$ then $N = 321$. What is the smallest number M such that $N = 3M$?