- No calculators are allowed.
- Pictures are only sketches and are not necessarily drawn to scale or proportion.
- You have one hour and fifteen minutes to complete the entire team session.

These 10 problems require exact numerical or algebraic answers. Exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers.

The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage in case of a tie.*

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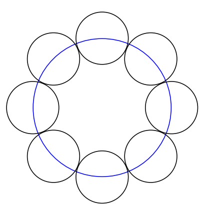
1. Eight circles of radius 1 have centers on a larger common circle and adjacent circles are tangent. Find the area of the common circle. See the illustration below.



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2. Suppose x and y are positive integers satisfying

$$x^3y + 35y^4 = 2018.$$

Find xy.

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3. What is the remainder if 10^{2018} is divided by 999?

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4. You roll a fair die five times and add the numbers that come up. What is the probability that the sum is 10?

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5. Consider the Fibonacci sequence: $a_1 = 1$, $a_2 = 1$, and $a_{n+2} = a_n + a_{n+1}$. What is the remainder when

$$\sum_{n=1}^{2018} a_n^2$$

is divided by 7.

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6. There are 46656 6-digit numbers that can be formed from the digits 1, 2, 3, 4, 5, and 6, with repetition of digits allowed. If these numbers are listed in order what is the 2018^{th} number in the list?

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7. A box contains a collection of 9 cent stamps and 14 cent stamps. The total value of the 9 cent stamps is twice the total value of the 14 cent stamps and the total value of all the stamps is less than $\$38.00$. What is the maximum number of stamps in the box.

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8. You are asked to go to the store and purchase apples, bananas, and oranges. Your mom says to buy a total of 20 pieces of fruit and your brother says that you must buy at least 1 of each fruit. How many ways can you make this purchase?

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9. Express the number

$$\sqrt[3]{207 + 94\sqrt{5}}$$

in the form $a + b\sqrt{5}$, with a and b integers.

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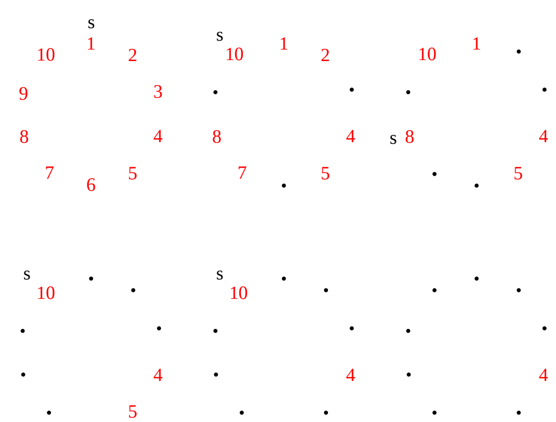
10. A series of n light poles are arranged in a circle and numbered, clockwise, from 1 to n. All the lights are initially on. You start at light pole $\#1$ and begin walking clockwise around the circle of lights turning off every third **lighted** pole doing so until there is only one pole that remains lighted. (To be clear, the first light turned off is $\#3$.) Let $f(n)$ be the number of the last remaining lighted pole. For example, below we illustrate that $f(10) = 4$. In each circle, the 's' represents your standing position and it is the third lit light you turn off. In the first circle all lights are on and you are standing at light $\#1$. In your first walk around you turn off the lights number 3, 6, and 9 and walk to the next lit light; you are now standing at light #10. In your second walk around you turn off light $\#2$ and $\#7$ and walk to the next lit light; you are now standing at light $\#8$. Continuing, you next turn off $\#1$ and $\#8$ and walk to the next lit light. You are now standing at light $\#10$ .You next turn off $\#5$ and walk to the next lit light. You are now standing at light $\#10$. Last, you turn off $\#10$ and walk to the next lit light. You are now standing at light $\#4$. The last light on is $\#4$, so $f(10) = 4$.



Find $f(2018)$.