- No calculators are allowed.
- Pictures are only sketches and are not necessarily drawn to scale or proportion.
- You have one hour and fifteen minutes to complete the entire team session.

These 10 problems require exact numerical or algebraic answers. Exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers.

The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.*

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1. George and Rick each have a bag of 12 marbles numbered 1 to 12. They each remove one ball uniformly at random from their own bag. Let g be the sum of the numbers on the balls remaining in George's bag and r be the sum of the numbers on the balls remaining in Rick's bag. Determine the probability that g and r differ by a multiple of 5.

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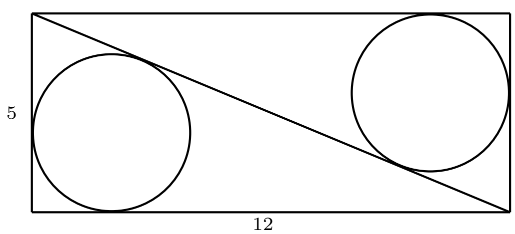
2. The rectangle below is 12×5. The diagonal forms two right triangles and a circle is inscribed in each triangle. Find the distance between the centers of the two circles.



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3. In the last century three people have birthdays in April and hold a common party the next month. They discover that if you multiply the ages of any two of them the product is the year in which the third person was born. In what year was the party held?

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4. A three digit number n is obtained by reversing the digits in the three digit number m. Their product is $nm = 548208$. Determine the smaller of n and m.

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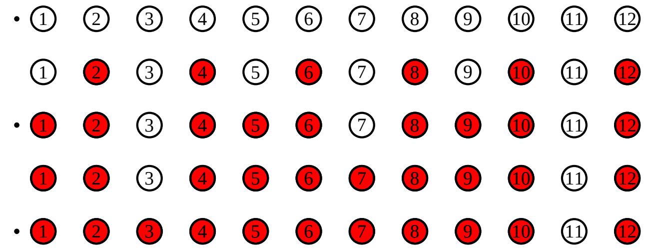
5. A school has a row of n open lockers, numbered 1 through n. Starting at the beginning of the row, you walk past and close every second locker until reaching the end of the row. Then you turn around, walk back, and close ever second locker that is still open. You continue in this manner back and forth along the row, until only one locker remains open. Define $f(n)$ to be the number of the last open locker. Find $f(2017)$.

   For example, if there are 12 lockers as illustrated below then $f(12) = 11$: After the first pass all the even numbered lockers are closed. After the second pass lockers 1, 5, and 9 are closed. After the third pass locker 7 is closed. After the fourth pass locker 3 is closed leaving locker 11 the remaining open locker. The $\cdot$ indicates the starting position at the beginning of each pass. A filled red circle indicates a closed locker.



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6. Find a polynomial $P(x)$ such that

   (a) $P(x+1) - P(x) = x^3$

   (b) $P(0) = 0$.

   Write your answer in the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0.$$

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7. Two rational numbers r and s are given. The numbers $r+s$, $r-s$, rs, and s/r are computed and rearranged in increasing order to get the list:

$$\frac{1}{12}, \frac{15}{16}, \frac{5}{3}, \frac{31}{12}.$$

   Find $r^2 + s^2$.

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8. Three integers a, b, and c have a product of 12012. Further $\frac{a}{4} = b + 2 = c - 8$. What is $a + b + c$?

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9. A standard fair 6-sided die is rolled 10 times. Given that the number 1 appears 4 times, what is the probability that no two 1's appear on consecutive rolls.

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10. Choose points D, E, and F on the sides of the equilateral $\triangle ABC$ so that $|AF| = |EC| = |DB| = 1$ and $|FB| = |DC| = |AE| = 3$. The line segments EB, AD, and CF enclose a triangle that is shaded in the diagram. Find the ratio of the area of the shaded region and the area of $\triangle ABC$.