Questions 1 - 21 are worth 1 point each and questions 22 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 21 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by writing your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1. The line that goes through $(1, 3)$ and $(k-1, k+4)$ has slope 2. Find k

A 1 B 2 C 3 D 4 E 5

2. You have 2 pairs of identical red socks, 2 pairs of identical green socks, 3 pairs of identical blue socks, and 4 pairs of identical yellow socks all stuffed in drawer. (A left sock is indistinguishable from a right sock.) One morning you rummage through the drawer and pull one sock out at time until you have a matching pair. How many socks must you pull out to guarantee having a matching pair?

A 2 B 3 C 5 D 8 E 11

3. Evaluate

$$\log_3(81^{10}).$$

A 3 B 30 C 4 D 40 E none of these

4. On a test the girls averaged 80 and the boys averaged 60. The overall class average is 76. What proportion of those taking the test are boys?

A .2 B .4 C .5 D .6 E .8

5. Suppose

$$f(x) = \frac{3x+4}{2x-1}.$$

Find $f^{-1}(2)$.

A 3 B 6 C $\frac{10}{3}$ D $\frac{3}{10}$ E 1

6. What is the area of the triangle with vertices $(1, 2)$, $(5, 2)$, and $(3, 6)$?

A 6 B 8 C 10 D 16 E none of these

7. How many six-digit multiples of 5 can be formed from the digits 1, 2, 3, 4, 5, and 6 using each digit exactly once?

A 24 B 32 C 64 D 120 E 720

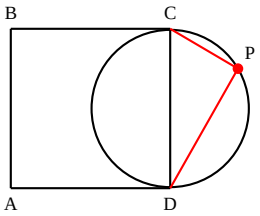
8. The number

$$\frac{(n-1)(n-3)(n-5)(n-7)-9}{2}$$

is an integer if n is

A 1 only B 2 only C 9 only
D any odd integer E any even integer

9. In the diagram, $ABCD$ is a square and P is a point on the circle with diameter CD. Suppose $CP = 7$ and $PD = 11$. What is the area of the square?



A 49 B 121 C 170 D 324 E none of these

10. What is the coefficient of x^7 in the polynomial $(x+4)^9$?

A 28 B 36 C 72 D 288 E 576

11. Let $x = \sqrt{7+2\sqrt{6}} + \sqrt{7-2\sqrt{6}}$. Which of the following intervals contains x?

A $(3.9, 4)$ B $(4.8, 4.9)$ C $(4.9, 5)$ D $(5, 5.1)$ E $(4.3, 4.4)$

12. The product of three consecutive positive integers is eight times their sum. What is the sum of their squares?

A 77 B 37 C 88 D 111 E 73

13. If $3^x = 11$ and $11^y = 27$ what is xy?

A 2 B 3 C 4 D 5 E 2.718

14. There are two urns containing candy. One urn contains two pieces: one green and one red. The other urn contains one green and three red pieces. You pick a piece of candy at random from each urn and eat the two chosen candies. If you ate exactly one piece of green candy, you draw another piece of candy from the urn still containing a green piece, and eat that. What is the probability that you ate two pieces of green candy?

A $\frac{1}{8}$ B $\frac{1}{4}$ C $\frac{3}{8}$ D $\frac{1}{2}$ E $\frac{5}{8}$

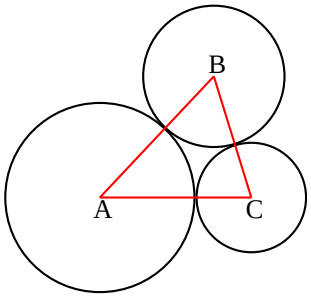
15. Which of the following numbers is a perfect square?

A $8! \cdot 9!$ B $9! \cdot 10!$ C $10! \cdot 11!$ D $12! \cdot 13!$ E $13! \cdot 14!$

16. What is the least value of $x + \frac{25}{x}$ if $x > 0$?

A 1 B 3 C 8 D 10 E 25

17. Three mutually tangent circles have centers A, B, and C. The lengths of the segments AB, BC, and AC are 21, 16, and 19, respectively. Find the radius of the circle with center B.



A 9 B 2 C 7 D 3 E 11

18. Compute $\log_2(3) \times \log_3(4) \times \log_4(5) \times \cdots \times \log_{63}(64)$.

A 1 B 6 C 128 D $\log_{63}(192)$ E $e^{64} - 1$

19. Two positive numbers a and b satisfy $2 + \log_2 a = 3 + \log_3 b = \log_6(a + b)$. Compute the value of $\frac{1}{a} + \frac{1}{b}$.

A 2 B 3 C 108 D 216 E 324

20. What is the unit digit of the integer 7^{2017}?

A 7 B 5 C 9 D 1 E 3

21. Eleven soccer teams play in a tournament. Each team must play each of the other teams exactly once. A winning team gets 5 points and the losing team gets 0 points. In the case of a tie each team gets 1 point. Let N be the total number of points awarded to all 11 teams. Which of the following is a possible value of N?

A 107 B 111 C 202 D 257 E 290

Questions 22 - 28 Exact Answers

These next seven questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out 20! or 6^{40}.

22. For what number k is the circle given by

$$x^2 + 8x + y^2 - 4y = k$$

tangent to the y-axis?

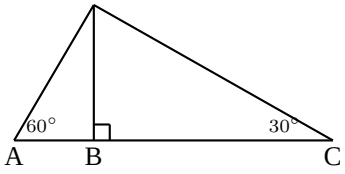
23. Solve the equation

$$8^{\frac{1}{6}} + x^{\frac{1}{3}} = \frac{7}{3 - \sqrt{2}}$$

for x.

24. A turtle starts moving west at 4 ft/min from a point P on the LSU Campus. Five minutes later, a snail starts moving south at 2 ft/min from a point 50 ft due east of P. What is the distance (in feet) between the two animals 5 minutes after the snail starts moving?

25. In the triangle given in the diagram below the length of AB is one. Find the length of AC.



26. In how many ways can 2017 be written as a sum of two or more consecutive positive integers written in increasing order? For example, $1008 + 1009$ would be counted but $1009 + 1008$ would not.

27. If k is a number such that

$$x^2 - 2017x + k$$

has 2015 as one root, what is the other root?

28. Suppose a and b are positive numbers and

$$a + b = ab = a^2 - b^2.$$

Find a.

Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 26**. Write your explanation on the **reverse side** of your answer sheet. *This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.*

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

It is graded only to separate first, second, and third place ties.