

Questions 1 - 21 are worth 1 point each and questions 22 - 28 are worth 2 points each.  
 No calculators are allowed.  
 Pictures are only sketches and are not necessarily drawn to scale or proportion.  
 You have one hour and twenty minutes to complete the entire morning exam.

**Questions 1 - 21 Multiple Choice**

Please:

- Use the answer sheet for your answers.
- Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.
- Completely erase any answer you wish to change.
- Do not make stray marks on the answer sheet.

1. Compute  $(505)^2 - (495)^2$ .  
 A 10    B 100    C 1000    D 10000    E none of these
2. If  $20x + 16y = 2016$  and  $16x + 20y = 1620$  what is  $x + y$ ?  
 A 1    B 100    C 101    D 1001    E none of these.

3. Suppose  $x$  is a negative real number. Then  $\frac{\sqrt{x^2}}{x}$  simplifies to  
 A  $-x$     B  $-1$     C 0    D 1    E  $x$
4. Suppose  $f(x) = 4x - 5$ . Solve  $f(f(x)) = 23$  for  $x$ .  
 A 3    B 5    C 7    D 23    E 25
5. How many quarts of antifreeze must be added to 8 quarts of water to create a 20% solution? Round your answer to the nearest tenth.  
 A 1    B 1.6    C 2    D 2.4    E 3

6. Simplify  $\frac{1}{\sqrt{7} + \sqrt{5}} + \frac{1}{\sqrt{7} - \sqrt{5}}$ .  
 A  $\frac{1}{2}$     B 2    C  $\sqrt{5}$     D  $\frac{\sqrt{35}}{2}$     E  $\sqrt{7}$

7. The sum of a positive number and 56 times its reciprocal is equal to its cube. What is the number?  
 A  $\sqrt{7}$     B  $-\sqrt{7}$     C 2    D  $2\sqrt{2}$     E 4
8. The average of 10 numbers is 8. The average of another 6 numbers is 16. What is the average of all 16 numbers combined?  
 A 8    B 9    C 11    D 15    E 16

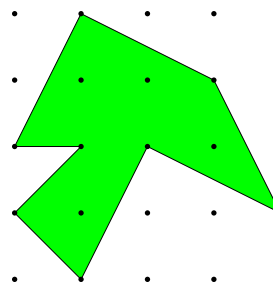
9. Plumber Joe was given the job to organize a yearly plumbing seminar. He was given a salary supplement of \$2016. He did such a great job that the next year his supplement was increased by 300%. The next year the seminar bombed so his supplement was then decreased by 75%. What is his salary supplement now?

A \$1512    B \$2016    C \$4536    D \$6048    E \$0

10.  $|x - 2| < 3$  if and only if  $a < x + 4 < b$ . Find the values of  $a$  and  $b$ .

A  $a = 6, b = 9$     B  $a = 3, b = 9$     C  $a = -5, b = 1$   
 D  $a = 0, b = 1$     E  $a = -3, b = 2$

11. The horizontal and vertical distance between two adjacent dots in the following diagram is 1. Find the area of the shaded region.



A 5    B 5.5    C 6    D 7    E 7.5

12. Rick flips a fair coin five times. Compute the probability that the fourth coin flip is the first time a heads comes up.

A  $\frac{1}{8}$     B  $\frac{1}{4}$     C  $\frac{3}{32}$     D  $\frac{1}{2}$     E  $\frac{1}{16}$

13. A beautiful mountain path takes you 6 hours to hike. One-third of the path is uphill, one-third is level, and one-third is downhill. Your average speed uphill is 2 miles per hour, your average speed on level land is 4 miles per hour, and your average speed downhill is 6 miles per hour. What is the length of the path? Round your answer to the nearest tenth of a mile.

A 1.5    B 2.0    C 15.3    D 19.6    E 21.4

14. Find the area of a rectangle whose perimeter is 50 ft and diagonal is 15 ft.

A 100    B  $100\sqrt{2}$     C 200    D  $200\sqrt{2}$     E 400

15. For all integer choices of  $n$  what value is  $(-1)^{n^4+n^2+1}$ ?

A 1    B  $-1$     C  $(-1)^n$     D  $(-1)^{n^2+1}$     E  $(-1)^{n^4+1}$

16.  $a + 2, 2a - 1, 4a - 10$  form the first 3 terms in an arithmetic progression. What is the fifth term?

A 8    B 3    C 14    D 31    E 20

17. The area of a certain equilateral triangle is equal to its perimeter. What is the length of one of its sides?

A  $\sqrt{3}$     B  $2\sqrt{3}$     C  $3\sqrt{3}$     D  $4\sqrt{3}$     E none of these

18. Suppose  $x_1$  and  $x_2$  are roots of  $x^2 - 2x - 2016$  and  $x_3$  and  $x_4$  are roots of  $x^2 - 5x - 2016$ . Compute  $x_4 - x_2 + x_3 - x_1$ .
- A -3      B -1      C 1      D 3      E 7

19. The following system has a solution  $(x, y)$ . Find  $x + y$ .

$$\frac{1}{x} - \frac{3}{y} = -1$$

$$\frac{4}{x} - \frac{9}{y} = 3$$

- A  $\frac{25}{42}$       B  $\frac{42}{25}$       C  $\frac{25}{3}$       D  $\frac{3}{25}$       E  $\frac{1}{3}$

20. A certain contest has 40 multiple choice questions and is scored in the following way: 5 points for each correct answer, 1 point for each question left blank, and 0 points for each incorrect answer. Arthur reports that he got a score of 195. Beth reports that she got a score of 189. Calvin says he got a 180, Donna says she got a 184. Finally, Edward reports that he got a 196. Who must be lying?

- A Arthur      B Beth      C Calvin      D Donna      E Edward

21. In order that

$$110893292387618678226N362326347776 = (2016)^{10}$$

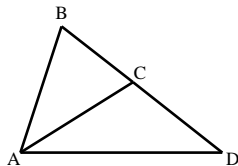
the letter  $N$  must be what digit?

- A 1      B 3      C 4      D 5      E 7

### Questions 22 - 28 Exact Answers

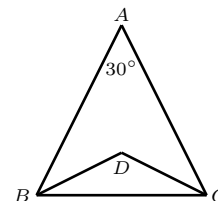
*These next seven questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for  $\pi$  or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out  $20!$  or  $6^{40}$ .*

22. In triangle  $\triangle ABD$  below  $AB = AC = CD$  and  $AD = BD$ . What is  $\angle ABD$  in degree measure?



23. It takes 654 digits to number the pages of a book that starts with page 1. How many pages are in the book?

24. In the diagram  $AB = AC$ ,  $BD$  bisects angle  $ABC$  and  $CD$  bisects angle  $ACB$ . Angle  $\angle BAC = 30^\circ$ . Find the angle  $\angle BDC$  (in degrees).



25. A die is a cube (with 6 sides) numbered 1 to 6. Two dice are rolled. What is the probability that the two numbers that come up are different?

26. There are two simultaneous solutions,  $(a, b)$  and  $(c, d)$ , to the equations

$$4x^2 + 9y^2 = 90$$

$$2x + 3y = 6.$$

Find  $a + b + c + d$ .

27. Find all ordered pairs  $(x, y)$  which satisfy the equation

$$(2x + y - 1)^2 + (-x + 2y + 8)^2 = 0.$$

28. Recall that a number  $N$  given in base  $b$  can be expressed in standard form  $[d_m d_{m-1} \dots d_0]_b$  and is such that  $N = d_m b^m + d_{m-1} b^{m-1} + \dots + d_1 b + d_0$ , where each digit  $d_i$  satisfies  $0 \leq d_i < b$ ,  $i = 0, 1, \dots, m$ . Compute

$$\frac{[222]_9}{[222]_3} \cdot [222]_4$$

and express your answer in base 8, standard form.

### Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 22**. Write your explanation on the **reverse side** of your answer sheet. *This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.*

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

**It is graded only to separate first, second, and third place ties.**