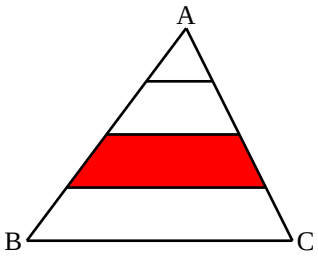
- No calculators are allowed.
- Pictures are only sketches and are not necessarily drawn to scale or proportion.
- You have one hour and fifteen minutes to complete the entire team session.

These 10 problems require exact numerical or algebraic answers. Exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers.

The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.*

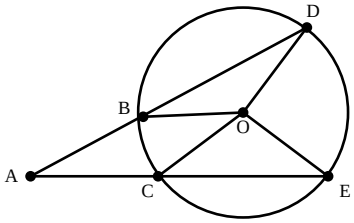
1. The digits 1, 2, 5, 6, and 8 can be reordered (without repetition) to form $5! = 120$ distinct 5-digit numbers. How many of these are divisible by 11?

2. In triangle ABC, three lines are drawn parallel to side BC dividing the altitude of the triangle into four equal parts. If the area of the second largest part (the shaded region) is 35, what is the area of the whole triangle ABC?



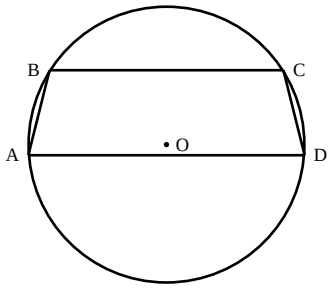
3. Three positive integers a, b, and c have the property that $a+b+c = 31$ and $abc = 1001$. Find $a^2+b^2+c^2$.

4. In the diagram below, B is on the line joining A and D, C is on the line joining A and E, $\angle BAC = 28°$, O is the center of the circle, and $\angle BOC = 30°$. Find $\angle DOE$.



5. A ball is dropped from a height of 20 feet onto a perfect flat surface. Each time the ball hits the surface after falling h feet it bounces a distance of $\frac{2}{5}h$ feet. Find the total distance the ball travels up and down if it is allowed to continue bouncing indefinitely.

6. In the diagram below A, B, C, and D are points on a circle of radius r and center O. Suppose $\overline{AD}$ is parallel to $\overline{BC}$, $AD = 26$, $BC = 22$, and $AB = CD = \sqrt{68}$. Find r.

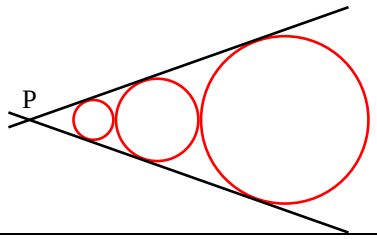


7. Let $\triangle ABC$ be an isosceles triangle with $\overline{AC}$ congruent to $\overline{BC}$. Suppose there is a point D between B and C such that the segments $\overline{AB}$, $\overline{AD}$, and $\overline{DC}$ have unit length. What is the length of line segment $\overline{AC}$?

8. Determine the number of pairs (x, y) of real numbers that satisfy the system of equations

$$x + xy + y = -9 \quad \text{and} \quad x^2 + y^2 = 17.$$

9. Three circles are externally tangent as shown in the figure below. The two lines through the point P are each tangent to all three circles. The radius of the largest circle is 4 and the radius of the smallest circle is 1. If θ is the acute angle formed by the two lines what is $\cos\theta$?



10. In triangle ABC suppose that the measure of angle C is two times the measure of angle A. If the length of $\overline{AB}$ is 6 units and the length of $\overline{CB}$ is 4 units, what is the length of $\overline{AC}$?