Questions 1 - 13 are worth 1 point each and questions 14 - 24 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

The people supervising this test are not permitted to explain to you the meaning of any question.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 13 Multiple Choice

Please:

- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1. If $x - y = 2$ and $x^2 - y^2 = 8$ what is $2x - 6y$?

A 0 B -16 C 16 D -14 E 14

2. $\sqrt{9 + 4\sqrt{5}}$ is the same as

A $2 + 3\sqrt{5}$ B $3 + 2\sqrt{5}$ C $1 + 2\sqrt{5}$ D $2 + \sqrt{5}$ E $3 + \sqrt{5}$

3. For a choice of the digit d given below the number

$$20150000201d$$

is a prime number. What is d?

A 1 B 3 C 5 D 6 E 7

4. How many integers between 1 and 2015, inclusive, are divisible by 6 or 9 but not both?

A 336 B 447 C 558 D 669 E 780

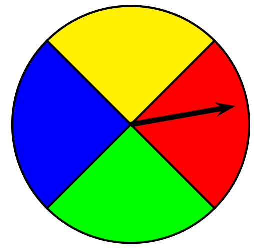
5. What is the area of the equilateral triangle that has a circumscribed circle with radius 10?

A $25\sqrt{3}$ B $50\sqrt{3}$ C $50\sqrt{2}$ D $75\sqrt{3}$ E 100π

6. The number 2.5 is split into two nonnegative real numbers x and y uniformly at random so that $x + y = 2.5$. Each number is then rounded to its nearest integer and added. For instance, if $x = 2.145$ then $y = .355$; x rounds to 2 and y rounds to 0; and $2 + 0 = 2$. What is the probability that the two rounded integers sum to 3?

A 0 B .4 C .5 D .6 E 1

7. The four colors, red, green, yellow, and blue on the spin wheel as illustrated below have equally likely probability on each spin. Suppose the wheel is spun four times. What is the probability that each color comes up?



A $\frac{1}{256}$ B $\frac{1}{64}$ C $\frac{3}{64}$ D $\frac{3}{32}$ E $\frac{3}{16}$

8. Evaluate

$$\log_{10} \tan 1^\circ + \log_{10} \tan 2^\circ + \log_{10} \tan 3^\circ + \cdots + \log_{10} \tan 89^\circ.$$

A 0 B 1 C $\frac{\pi}{2}$ D $\log_{10} \frac{\pi}{2}$ E none of these

9. How many subsets of $\{a, b, c, d, e, f, g\}$ contain a and b but not c?

A 6 B 16 C 24 D 32 E 128

10. When 3^{2015} is divided by 13 what will be the remainder?

A 3 B 5 C 7 D 9 E 11

11. Jack has a bag of marbles. He gives Andrew half of his marbles and two more. He then gives Bob half of his marbles and two more. Finally, he gives Charlie half of his marbles and two more. Jack now just has one marble remaining in his bag. How many marbles did he give Bob?

A 8 B 9 C 10 D 20 E 22

12. If $x + \dfrac{1}{x} = 5$ what is $x^3 + \dfrac{1}{x^3}$?

A 110 B 120 C 125 D 130 E 135

13. Which of the following statements are true about

$$f(x) = \log(\sqrt{x^2 + 1} - x)?$$

I. The domain of $f(x)$ is all real numbers.
II. The graph of $f(x)$ contains the origin $(0, 0)$.
III. For all x, $f(-x) = -f(x)$.

A I only B I and II only C I and III only D II and III only E I, II, and III

Questions 14 - 24 Exact Answers

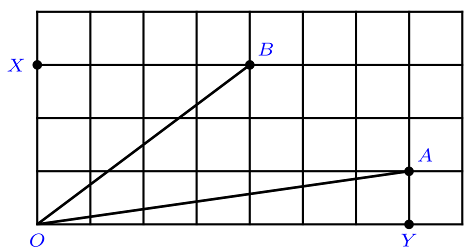
These next ten questions require exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40}.

14. If $18!$ is written in base 12 how many zeros would be at the end?

15. A linear function f satisfies $f(1) = 3$ and $f(f(1)) = 7$. Find $f(f(2))$.

16. In a group of 4 friends the sums of the ages of any 3 of them are 45, 36, 40, and 41. What is the age of the oldest?

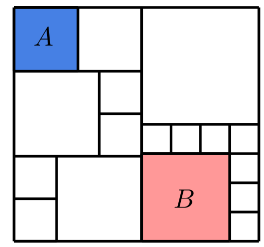
17. In the unit-grid below $\alpha = \angle AOY$ and $\beta = \angle BOX$. Find $\beta - \alpha$. Express your answer in degrees.



18. A rectangular solid has a top face with area 56 square units and two side faces with areas 21 and 24 square units. What is the volume of the solid (in cubic units)?

19. For how many positive integers n is $1! + 2! + 3! + \cdots + n!$ a perfect square, i.e., the square of an integer?

20. The following rectangular region is made from squares with squares A and B highlighted. Find the ratio of side A to side B.



21. A box contains 6 red balls, 4 blue balls, and 2 green balls. Three balls are randomly drawn (without replacement). What is the probability that the three balls selected are all the same color?

22. Boudreaux and Thibodeaux play a game consisting of 8 rounds. After the n^{th}-round, $n = 1, 2, \ldots, 8$, the winner receives 2^{n-1} dollars from the loser. Suppose Boudreaux plays a single game and finishes ahead of Thibodeaux by 35 dollars. Determine the sum of the rounds that Boudreaux wins. For example. if Boudreaux wins rounds 2, 4, and 6 your answer would be 12 $(2 + 4 + 6)$.

23. My two-digit age (in years) is one short of a perfect square. The product of the digits in my age is my wife's age. The sum of the digits in my age is my son's age who is seven years older than my daughter. Her age is the sum of the digits in my wife's age. How old is my wife?

24. If $\triangle ABC$ has sides 2, 5, and 6 find $\cos\angle A + \cos\angle B + \cos\angle C$.

Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 20**. Write your explanation on the **reverse side** of your answer sheet.

This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

It is graded only to separate first, second, and third place ties.