Questions 1 - 21 are worth 1 point each and questions 22 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 21 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1. The expression $\frac{1}{2} - \frac{1}{4} - \frac{1}{3} + \frac{1}{6}$ is the same as

A $\frac{1}{3}$ B $\frac{2}{3}$ C $\frac{1}{4}$ D $\frac{3}{4}$ E $\frac{1}{12}$

2. How many negative roots are there to the equation

$$x^4 - 2x^3 + 3x^2 - 4x + 5 = 0?$$

A 0 B 1 C 2 D 3 E 4

3. If $xy = 4$ and $x^2 - y^2 = 9$ then $\frac{2x}{y} - \frac{2y}{x} =$

A $\frac{9}{2}$ B 18 C $\frac{9}{4}$ D $\frac{2}{9}$ E 2

4. Ann's number is added to Bob's number and it is observed that the sum is the same as the product. Bob's number is then multiplied by one less that Ann's number. What can we conclude about this new product:

A The product is negative.
B The product is smaller that Bob's number.
C The product is an integer.
D The product is the same as Ann's number.
E The product must be zero.

5. How many positive integers are divisors of 2015

A 4 B 5 C 6 D 7 E 8

6. A linear function f satisfies $f(1) = 5$ and $f(x+1) = f(x) + 3$. Find $f(20)$.

A 43 B 62 C 4 D 34 E 26

7. What is the sum of the digits to the integer solution to

$$\sqrt{5 - \sqrt{15 - \sqrt{x - 2015}}} = 2?$$

A 3 B 4 C 5 D 6 E 8

8. There are two roots to the equation $x^2 - 4x + 1 = 0$. Find their sum.

A $4 + \sqrt{12}$ B 4 C -4 D $-4 + \sqrt{12}$ E 0

9. You have some nickels, dimes, and quarters that total \$2. You have 2 more quarters than dimes and 3 times as many nickels as dimes. How many coins do you have?

A 3 B 5 C 11 D 17 E 21

10. If $9^{x+1} - 9^x = 216$ what is $\left(\frac{8x}{3}\right)^x$?

A 2 B 4 C 6 D 8 E 10

11. Find the smallest positive integer n that satisfies

$$\frac{2}{5} + \frac{3}{n} < \frac{1}{4} + \frac{1}{3}.$$

A 13 B 14 C 15 D 16 E 17

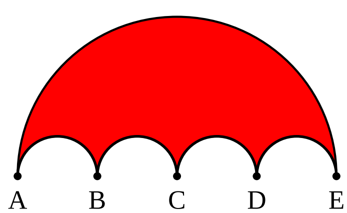
12. What is the area of the square whose circumscribed circle has radius 10?

A 40 B $100\sqrt{2}$ C 200 D $200\sqrt{2}$ E 100π

13. Suppose a and b are real numbers such that $17^a = 16$ and $17^b = 4$. What is $2^{\frac{1}{a-b}}$?

A 17 B $\frac{1}{17}$ C $\sqrt{17}$ D $\frac{1}{\sqrt{17}}$ E 1

14. The points A, B, C, D, and E are collinear and $AB = BC = CD = DE = 1$. The curves are semicircles. What is the area of the shaded region?



A $\frac{\pi}{2}$ B π C $\frac{3\pi}{2}$ D 2π E $\frac{5\pi}{2}$

15. How many positive integers less than 1000 have the sum of their digits equal to 5?

A 15 B 21 C 23 D 25 E 27

16. If $x + \frac{1}{x} = 5$ what is $x^2 + \frac{1}{x^2}$?

A 25 B 23 C 21 D 19 E none of these

17. Suppose a, b, c, and d are positive real numbers such that $\frac{a}{b} < \frac{c}{d}$. Which of the following inequalities (when placed in the spaces) satisfy the following:

$$\frac{a}{b} \;\underline{\quad}\; \frac{a+c}{b+d} \;\underline{\quad}\; \frac{c}{d}?$$

A $<, <$ B $<, >$ C $>, <$ D $>, >$ E none of these

18. What is the 2015^{th} digit in the sequence of integers starting at 1? I.e. $1234567891011121314\ldots$.

A 0 B 1 C 3 D 8 E 9

19. Suppose $a = \dfrac{1}{\sqrt{7}-\sqrt{6}}$ and $b = \dfrac{1}{\sqrt{7}+\sqrt{6}}$. Find

$$ab^3 + a^2b^2 + ba^3.$$

A 19 B 26 C 27 D 34 E 0

20. Suppose x and y are two real numbers that satisfy $xy = 6$ and $x^2y + xy^2 + x + y = 63$. Find $x^2 + y^2$.

A 93 B 36 C 144 D 75 E 69

21. A cube is made of solid white material but painted red on the outside. It is then cut into 125 equally sized smaller cubes. How many of these smaller cubes have red painted on just one side?

A 9 B 25 C 36 D 54 E 96

Questions 22 - 28 Exact Answers

These next seven questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40}.

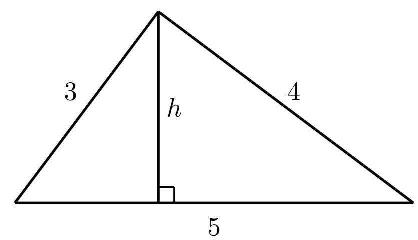
22. Your school is raffling a \$300 bicycle. Tickets are sold at 75 cents a piece. How many tickets must be sold for the school to make a profit of \$60?

23. Find the largest integer n so that 2^n divides $20!$.

24. Suppose a set with 70 elements is divided into n sets each with $9m + 8$ elements, where m is a positive integer. What is m?

25. Find the smallest sum of two positive integers n and m so that $nm = 2015$

26. What is the height h of the 3-4-5 right triangle indicated below:



27. How many pairs of positive integers (n, m) are there such that $2n + 3m = 2015$?

28. Suppose f is a function defined for all real numbers and

$$f(x) + 2f(3-x) = x^2,$$

for all x. What is $f(1)$?

Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 25**. Write your explanation on the **reverse side** of your answer sheet. *This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.*

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

It is graded only to separate first, second, and third place ties.