Questions 1 - 13 are worth 1 point each and questions 14 - 24 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

The people supervising this test are not permitted to explain to you the meaning of any question.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 13 Multiple Choice

Please:

- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1. Suppose x satisfies $3\log_5 x = \log_5 8$. Find x.

   A 1 B 2 C 5 D 8 E none of these

2. An urn contains only blue and red marbles. If you randomly select 6 marbles at a time at least one of them will be red and if you randomly select 9 marbles at a time there will be at least one red and one blue. What is the maximum number of marbles in the urn?

   A 10 B 13 C 14 D 15 E 16

3. How many pairs of integers (x, y) are there such that $1 < x < y$ and $xy = 2014$?

   A 0 B 1 C 2 D 3 E more that 4

4. The parabola $y = 1 - ax^2$ intersects the coordinate $x-$ and $y-$axes in three points which form the vertices of an equilateral triangle. Determine the value of a.

   A 1 B $\sqrt{3}$ C 2 D $2\sqrt{3}$ E 3

5. How many two digit prime numbers are there such that each digit is a prime number?

   A 0 B 1 C 4 D 16 E none of these

6. Boudreax, Thibideaux, and Maria are the only applicants for a job at the local crawfish cannery. Boudreax and Thibideaux are equally likely to get the job but Maria's chance of being hired is 20% higher than Thibideaux's due to her superior work skills. What is the probability that Maria will be hired.

   A .625 B .2 C .3125 D .375 E .5

7. The sum of the lengths of 3 sides of a rectangle is 2014. The sum of the lengths of the fourth side and the diagonal is also 2014. Determine the ratio of the length of the longer side to the length of the shorter side of this rectangle.

   A $\sqrt{2}:1$ B $\sqrt{3}:1$ C $2:1$ D $3:1$ E $4:1$

8. Determine the unit digit in the number

   $$(2014)^{((22)^3)}.$$

   A 2 B 4 C 6 D 8 E 0

9. What is the value of the sum

   $$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{99}{100!}.$$

   A $\frac{1}{2} - \frac{1}{100!}$ B $\frac{1}{2} - \frac{1}{99!}$ C $1 - \frac{1}{100!}$ D $1 - \frac{1}{99!}$ E 1

10. What is the smallest integer n such that $\frac{100!}{(50)^n}$ is not an integer.

    A 11 B 12 C 13 D 14 E 15

11. There are several dogs and people in a dog park. It is noted that there are 23 heads and 84 feet. How many dogs are there?

    A 19 B 4 C 7 D 23 E 8

12. Suppose $f(g(x)) = x^4$ and $f(x) = (x+1)^2$. Which of the following could be $g(x)$?

    A $x^2 - 1$ B $x^2 + 1$ C $x^4 - 1$ D $(x+1)^4$ E $\frac{x^4}{(x+1)^2}$

13. How many ordered pairs of integers (x, y) satisfy $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$?

    A 2 B 4 C 5 D 6 E 7

Questions 14 - 24 Exact Answers

These next ten questions require exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40}.

14. Suppose a, b, and c are distinct numbers and

    $$a^2 - bc = 7, \qquad b^2 + ac = 7, \qquad c^2 + ab = 7.$$

    Find $a^2 + b^2 + c^2$.

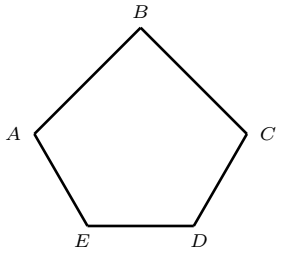
15. Suppose a polynomial, $p(x)$, has integer coefficients. The remainder when $p(x)$ is divided by $x - 3$ is 22 and the remainder when $p(x)$ is divided by $x - 22$ is 3. If $r(x)$ is the remainder when $p(x)$ is divided by $(x - 3)(x - 22)$, find $r(2014)$.

16. How many integers x are there in the set $\{1, 2, 3, \ldots, 2014\}$ that satisfy $x^3 - x^2$ is a perfect square.

17. Three balls are stacked in a cylinder that touches the stack on all sides, on the top and on the bottom. Find the ratio of the volume of the balls to the volume of the cylinder.

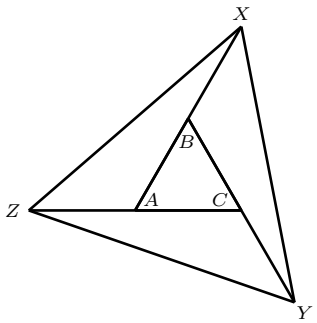
18. If you were to expand $(x + y)^7$ and add up all the coefficients what would you get?

19. The pentagon $ABCDE$ has a right angle at $\angle ABC$. The lengths of sides AB and BC are $\sqrt{2}$ while the lengths of sides AE, ED, and DC are 1. Also, angles $\angle AED$ and $\angle EDC$ are equal. Find the area of the pentagon.



20. A deranged university professor takes his bag of 20 marbles to the top of Tureaud Hall where he drops them one by one. One minute after dropping the first marble he drops the second one. Two minutes after dropping the second marble he drops the third one. Three minutes after dropping the third marble he drops the fourth one. He continues in this manner and at $6:00$ p.m. he drops the 20th marble. At what time did he begin losing his marbles? (Express your answer in the form h:m, for example 2:35)

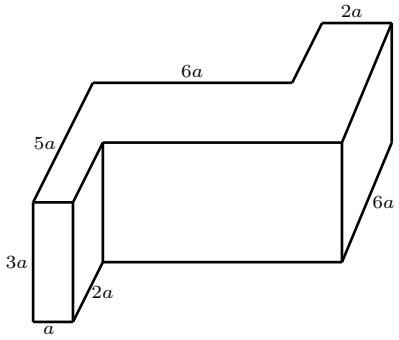
21. *(This is also the tie breaker question)* In the diagram below $\triangle ABC$ is an equilateral triangle with area 1. Further, B is the midpoint of AX, C is the midpoint of BY, and A is the midpoint of ZC. What is the area of $\triangle XYZ$?



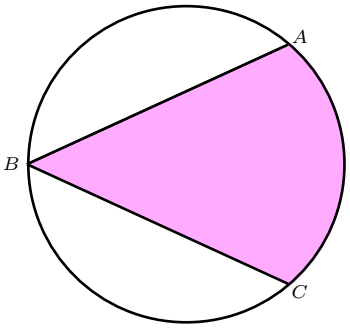
22. Find the sum of all solutions to

$$\frac{1}{x^2 - 2x + 3} + \frac{1}{x^2 - 2x - 1} = \frac{2}{x^2 - 2x}.$$

23. Find the surface area of the solid in the diagram below. (All corner angles are $90°$.)



24. In the diagram below $\angle ABC$ is $30°$. Find the area of the shaded region in the circle of radius 1.



Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 21**. Write your explanation on the **reverse side** of your answer sheet.

This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

It is graded only to separate first, second, and third place ties.