Questions 1 - 21 are worth 1 point each and questions 22 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 21 Multiple Choice

Please:

- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1. Simplify $\dfrac{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}+\frac{1}{5}}$.

   A -3 B $\frac{25}{17}$ C -1 D $\frac{3}{7}$ E $\frac{10}{3}$

2. Determine the y-intercept of the line that passes through $(10, 2)$ and $(8, 5)$.

   A 0 B -13 C $\frac{26}{3}$ D 17 E -7

3. Simplify the following expression:

$$3-(8)(11-11)^4+7\left(\frac{15-7}{100-6}\right)^0.$$

   A -5 B 3 C 10 D 2 E none of these

4. Suppose $a=\dfrac{3}{7}$, $b=\dfrac{9}{22}$, and $c=\dfrac{21}{50}$. Then

   A $a<b<c$ B $a<c<b$ C $b<a<c$
   D $b<c<a$ E $c<a<b$

5. A dress costs \$151.20 including tax. If the tax rate is 8% what is the amount of the tax.

   A \$11.20 B \$140.00 C \$8.00 D \$10.20 E \$9.80

6. The expression $\dfrac{a^3b^{-2}(cb)^3}{c^{-4}b^2(ac)^3}$ is equivalent to

   A 1 B b^3 C $\frac{a^3c^3}{b^4}$ D $\frac{c^4}{b}$ E none of these

7. Which of the following is an equation of the line that goes through the points $(3, 4)$ and $(15, 20)$.

   A $y=4x+4$ B $y-4=\frac{3}{4}(x-3)$ C $y=\frac{3}{4}x$
   D $y=3x+15$ E none of these

8. The radius of the circle given by the equation

$$x^2+y^2+8x-10y+5=0$$

   is

   A 3 B 4 C 5 D 6 E 7

9. When 270 is divided by an odd number the quotient is a prime number and the remainder is 0. What is the quotient?

   A 2 B 3 C 5 D 7 E 11

10. Jack and Jill their first test did miss;
    the rest of the class averaged seventy-six.
    On make-up Jack got a perfect score;*
    Jill struggled to finish with eighty-four.
    On return the class thought it such a bore
    when told that the average had increased by four.

    How many students (including Jack and Jill) are in this class?

    * I.e. 100

    A 8 B 11 C 18 D 25 E 26

11. Simplify $\dfrac{10^6-10^4}{99}$.

    A $\frac{1}{99}$ B $\frac{100}{99}$ C 10^2 D 10^3 E 10^4

12. Which of the following numbers is the largest?

    A 1 B $2^{\frac{1}{2}}$ C 1.2 D $3^{\frac{1}{3}}$ E $5^{\frac{1}{5}}$

13. Determine the value of m so that the three lines

$$\begin{aligned} y &= mx+8 \\ y &= 2x-4 \\ y &= 3x \end{aligned}$$

    intersect at one point.

    A 1 B 2 C 3 D 4 E 5

14. A certain business opens at 1pm and closes just before 12 midnight. The business building has 6 doors. The owner has a mischievous teenager who has programmed the locks on the doors to be open only at certain hours. Door 1 is open only at odd hours. Door 2 is open only at even hours. Door 3 is open at hour n if n is prime. Door 4 is open at hour n if n is divisible by 3. Door 5 is open at hour n if n is a perfect square. Door 6 is open at hour n if n is a Fibonacci number. At what hour will most doors be open? (To be clear, a door open at, for example, hour 2 means that door is open from 2 pm to just before 3 pm.)

    A 1 B 3 C 4 D 6 E 11

15. For n an integer what does $(-1)^{(n^4-3n+2)}$ simplify to?

    A 1 B -1 C $(-1)^n$ D $(-1)^{n^2}$ E none of these

16. What is the least integer n such that $|n-2014|<22$?

    A 2014 B 2033 C 1992 D 1993 E 2034

17. The graph of $|x| + |y| = 4$ is a

A circle B square C triangle D pentagon E none of these

18. Suppose x and y are non negative real numbers such that $x^2 + y^2 = 5$ and $xy = 7$. Determine $x + y$.

A 12 B $\sqrt{2}$ C $\sqrt{19}$ D 2 E $\sqrt{12}$

19. If $\dfrac{a+3b}{a-3b} = 7$ then what is $\dfrac{a+2b}{a-2b}$?

A 1 B 3 C 5 D 7 E none of these

20. A rectangle has perimeter 24 and one side of length 4. What is its area?

A 32 B 16 C 8 D 24 E none of these

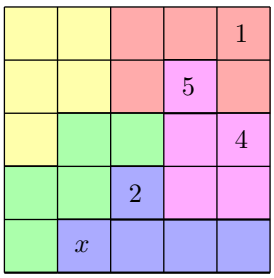
21. Find the number of digits in $4^{15}5^{25}$ when written in decimal expansion.

A 25 B 26 C 27 D 28 E 29

Questions 22 - 28 Exact Answers

These next seven questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out 20! or 6^{40}.

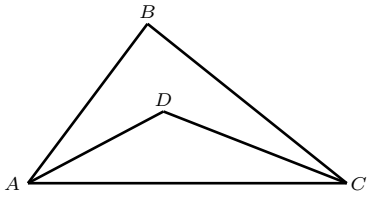
22. The following colored grid may be filled with the numbers 1, 2, 3, 4, and 5 so that each row, column, and colored region contains these numbers exactly once. Find x.



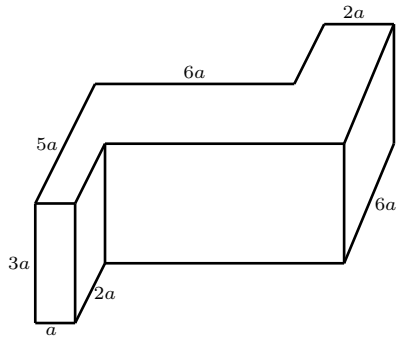
23. Find all solutions to $3^x + 3^{x+1} + 3^{x+2} = 351$.

24. Square A has perimeter 8 and square B has diagonal 3. What is the length of the side of square C if its area is the sum of the areas of square A and square B.

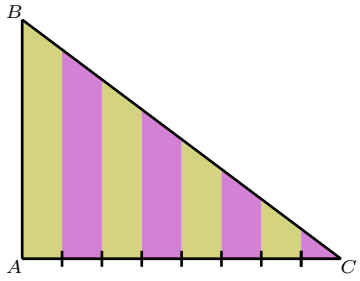
25. *This is also the tie breaker question* In triangle $\triangle ABC$ the measure of $\angle ABC$ is $100°$. Point D is chosen in the triangle so that line DA bisects $\angle BAC$ and DC bisects $\angle ACB$. Find the degree measure of $\angle ADC$.



26. Find the volume of the solid in the diagram below. (All corner angles are $90°$.)



27. In triangle $\triangle ABC$ the lengths of sides AC, AB, and BC are 8, 6, and 10, respectively. Find the ratio of the area of the region shaded gold to the area of the region shaded violet in the following triangle. The divisions along AC divide that side into unit lengths.



28. A positive two-digit number is called **frothy** if it is equal to four times the sum of its digits. Find the sum of all frothy two-digit numbers.

Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 25**. Write your explanation on the **reverse side** of your answer sheet. *This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.*

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

It is graded only to separate first, second, and third place ties.