Questions 1 - 12 are worth 1 point each and questions 13 - 24 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

The people supervising this test are not permitted to explain to you the meaning of any question.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 13 Multiple Choice

Please:

- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

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1. Suppose $p(x)$ is a polynomial of degree two, has roots -2 and 4, and $p(0) = 24$. What is the coefficient of x in $p(x)$?

   A -6 B -2 C 0 D 2 E 6

2. The number $\sqrt{9-3\sqrt{5}}+\sqrt{9+3\sqrt{5}}$ simplifies to

   A $\sqrt{30}$ B 6 C$\sqrt{90}$ D18 E $27\sqrt{5}$

3. Find the unit digit in the decimal expansion of the number $3^{\left(2^{2013}\right)}$.

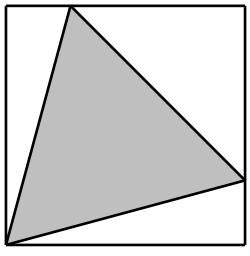
   A 1 B 3 C 7 D 9 E None of these

4. Find the angle (in degrees) between the hour hand and the minute hand on a normal 12-hour analog clock at 3:02.

   A 74 B 79 C 84 D 89 E None of these

5. An equilateral triangle is inscribed in a square of side length 1 with a vertex of the triangle coinciding with a corner of the square as shown below. Find the area of the triangle.

   A $3-2\sqrt{2}$ B $\frac{1}{2}$ C $2\sqrt{3}-3$ D $\frac{2}{5}$ E $\frac{\sqrt{3}}{4}$



6. Let x be a real number. Find the minimum value of

$$|x-1|+|x-2|+\cdots+|x-7|+|x-8|\,.$$

   A 16 B 14 C 28 D 15 E 12

7. If Boudreaux rolls a standard six-sided die until he rolls the same number twice in a row, what is the probability that his 5^{th} roll is his last?

   A $1/2$ B $\frac{5^5}{6^5}$ C $\frac{5^3}{6^4}$ D $\frac{5^4}{6^5}$ E None of these

8. If $\sin x+\cos x=\frac{-1}{2}$ then the value of $\sin^3 x+\cos^3 x$ is

   A $\frac{-1}{8}$ B $\frac{7}{16}$ C $\frac{1}{2}$ D $\frac{-11}{16}$ E $\frac{9}{16}$

9. A five-digit number N is formed by placing the digits $1, 2, 3, 4$ and 5 in a random order. What is the probability that N is divisible by 6?

   A $1/6$ B $2/5$ C $1/2$ D $3/5$ E $5/6$

10. Find the number of positive integers n so that $2013+n^2$ is the square of a positive integer.

    A 1 B 2 C 3 D 4 E None of these

11. If the number 2013 is written in the form

$$1-2+3-4+5-6+\cdots+(n-2)-(n-1)+n$$

    then the sum of the digits in the decimal representation of n is

    A 2 B 11 C 4 D 10 E 7

12. Let n be the 3 digit number whose hundreds' digit is 3, tens' digit is x, and unit digit is 7. Thus $n = 3x7$. Similarly, let m be the three digit number $m = 26y$. How many pairs (x, y) of digits are there such that 33 divides the sum $n + m$.

    A 0 B 1 C 2 D 3 E 4

13. You are sent to a shop to purchase 12 doughnuts. The shop makes 3 types of doughnuts. How many different ways can you make your purchase assuming there is enough of each type available?

    A 220 B 36 C 301 D 144 E 91

Questions 14 - 24 Exact Answers

These next ten questions require exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40}.

14. What is the base 3 representation of 2013?

15. Find an integer c that satisfies the following system of equations.

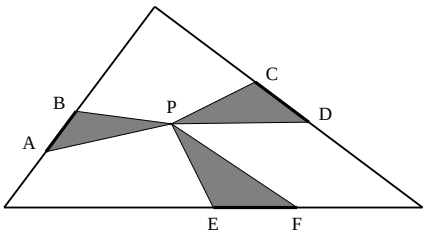
$$\begin{aligned} a+b+c &= 9 \\ abc &= 10 \\ c^2-a^2-b^2 &= 13 \end{aligned}$$

16. Let a and b be distinct real numbers satisfying

$$\frac{a}{b}+\frac{a+3b}{b+3a}=2.$$

Find a/b.

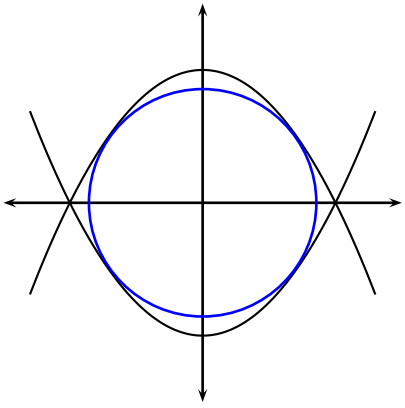
17. The triangle below is a $30, 40, 50$ triangle. Suppose A and B are the endpoints of a line segment of length 6 on the side of length 30, C and D are the endpoints of a line segment of length 8 on the side of length 40, and E and F are the endpoints of a line segment of length 10 on the side of length 50. Let P be any point inside the triangle. Find the sum of the areas of triangles $\triangle APB$, $\triangle CPD$, and $\triangle EPF$.



18. The two roots of the following quadratic equation are prime numbers. Find the value of c.

$$x^2-33x+c=0$$

19. Let R be the region bounded by the parabolas $y=1-x^2$ and $y=x^2-1$. Find the largest positive number r so that the circle with radius r and center $(0,0)$ lies in R. The following diagram may be helpful:



20. There is a non integer solution x to the following equation. Find it.

$$\frac{3}{2}+\log_3(x^{\frac{1}{4}})=\log_x(9x).$$

21. Two bicyclists are travelling toward each other on a straight road. One is travelling at 12 miles per hour (mph) and the other is traveling at 14 mph. When they are exactly 2 miles apart a mighty super bee begins travelling at 50 mph. She starts at one bicyclist's nose and flies to the other bicyclist's nose where upon she instantly turns around to fly to the other bicyclist's nose, and so forth, until the bicyclists meet. How far does the bee travel?

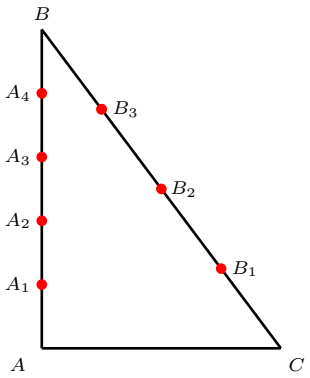
22. Suppose a function f is given by the following table

x	1	2	3	4	5
$f(x)$	2	4	1	5	3

Suppose $x_0=2$ and $x_{n+1}=f(x_n)$. What is the value of x_{2013}?

23. A sequence is defined by $a_{n+3}=a_{n+2}+2a_{n+1}+3a_n$, $n\geq 0$. Suppose $a_7=135$, $a_5=25$ and $a_4=9$. Find a_0.

24. Triangle $\triangle ABC$ has a right angle at A. The length of AB is 4 and the length of AC is 3. The points A_1, A_2, A_3, and A_4 partition AB into 5 segments of equal length. The points B_1, B_2, and B_3 partition BC into 4 segments of equal length. Let x be the area of $\triangle A_4B_3A_2$ and y the area of $\triangle A_3B_1A_1$. Find $\frac{x}{y}$.



Tie Breaker requiring Full Solution

Please give a **detailed explanation** of your solution to **Question 24**. Write your explanation on the **reverse side** of your answer sheet.

This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

It is graded only to separate first, second, and third place ties.