Questions 1 - 21 are worth 1 point each and questions 22 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

**Questions 1 - 21 Multiple Choice**

*Please:*
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1. 170% of 25 is

   A 35  B 40  C 42.5  D 48  E 54

2. $(4 \times 10^{65})(6 \times 10^{-33}) =$

   A $2.4 \times 10^{32}$  B $2.4 \times 10^{33}$  C $2.4 \times 10^{34}$
   D $24 \times 10^{31}$  E $240 \times 10^{32}$

3. If 40 grams of a certain perfume costs $12, how much will 70 grams cost?

   A $16  B $18  C $18.75  D $21  E $22

4. If $2^{33} = 8^x$, then $x =$

   A 4  B $\sqrt{66}$  C 11  D 16  E 22

5. Solve for $x$ and $y$:

   $$2x + y = 7$$
   $$4x - y = 11$$

   A $x = 3, y = 1$  B $x = 0, y = 7$  C $x = 1, y = 5$
   D $x = 2, y = 2$  E $x = 2, y = -3$

6. If the sum of three consecutive integers is at most 24, then the smallest of the three integers

   A  must be less than 6

   B  must be greater than 5

   C  must be 7

   D  can be no greater than 7

   E  is greater than 7

7. If $a < 0$ and $b < 0$, then

   A $a - b > 0$  B $ab < 0$  C $ab = 0$
   D $a < b$  E $ab > 0$

8. How many solutions does the equation $|x + 1| + 5 = 3$ have?

   A 0  B 1  C 2  D 3  E 4

9. $\dfrac{1}{2} + \dfrac{2}{3} =$

   A $\dfrac{1 + 2}{2 + 3}$  B $\dfrac{3 + 4}{2 \times 3}$  C $\dfrac{1 + 2}{2 \times 3}$  D $\dfrac{1 \times 2}{2 \times 3}$  E $\dfrac{3 + 2}{2 \times 3}$

10. If $ab = 0$, then

    A  $a = 0$

    B  $b = 0$

    C  both $a$ and $b$ must be zero

    D  at least one of $a$ or $b$ must be zero

    E  $a = -b$

11. Which of the following is not a root of

    $$(x - 2)(x - 1)(x + 5)(x + 7) = 0 ?$$

    A $-7$  B $-5$  C $1$  D $2$  E $4$

12. If $x \neq -1$, then $(x + 1)^0 =$

    A $x + 1$  B $x$  C $1$  D $0$  E $\dfrac{1}{x + 1}$

13. Which of the following *could not* represent two consecutive integers?

    A  $x, \ x + 1$

    B  $x, \ x - 1$

    C  $x - 8, \ x - 7$

    D  $x - \dfrac{1}{2}, \ x + \dfrac{1}{2}$

    E  $x - 1, \ x + 1$

14. If $n$ dimes and $n + 7$ quarters total at least $14.80, then

    A  $n + (n + 7) \geq 14.80$

    B  $n + (n + 7) \geq 1480$

    C  $10n + 25(n + 7) \geq 1480$

    D  $.10n + .25(n + 7) \geq 1470$

    E  $25n - 1480 = 10n$

15. In the figure below, four squares have been placed side by side to form a rectangle of perimeter 140. What is the area of each square?
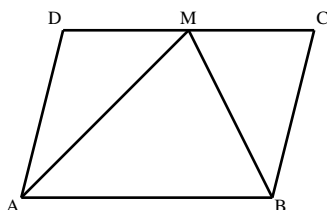


    A 400  B 360  C 300  D 256  E 196

16. In the figure, $PQRS$ is a parallelogram. $PT$ is drawn and extended to meet $SR$ at $U$. If the area of triangle $PQT$ is 7, what is the area of triangle $RTU$ ?
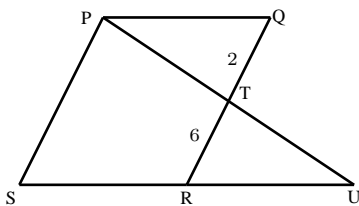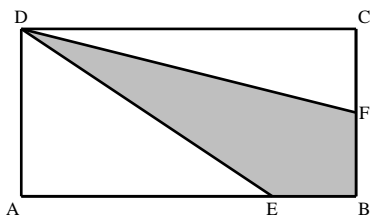


A 21          B 25          C 35          D 50          E 63

17. In the figure, $ABCD$ is a parallelogram, $\angle BAD = 60°$, $AM$ and $BM$ are angle bisectors of angles $BAD$ and $ABC$ respectively. If the perimeter of $ABCD$ is 6 cm, find the length of side $AB$.



A 1          B $\sqrt{2}$          C $\sqrt{3}$          D 2          E 3

18. [*This is also a tie breaker question* − see page 3.] In the rectangle, $CF = BF$ and $4BE = AE$. What is the ratio of the shaded area to the non-shaded area?



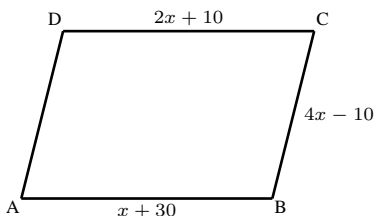A $\frac{1}{3}$          B $\frac{7}{20}$          C $\frac{3}{8}$          D $\frac{7}{13}$          E $\frac{13}{20}$

19. $\sqrt[4]{32} \div \sqrt[4]{2} =$

A 2          B 4          C 8          D 16          E 24

20. A mechanic can completely service a car in $1\frac{2}{5}$ hours. If he works from 8 AM to 5 PM on his job with one hour off for lunch, how many cars can he completely service in one working day?

A 4          B 5          C 6          D 7          E 8

21. In the figure, $ABCD$ is a parallelogram.



The length of $AD$ is

A 20          B 50          C 54.5          D 60          E 70

**Questions 22 - 28 Exact Answers**

*These next seven questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for $\pi$ or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or $6^{40}$.*

22. The streets in a city form a grid, running north-south or east-west. Alice, Bob, Carol, and Don live at four street intersections which are vertices of a rectangle, with Alice and Don at opposite vertices. The school that they all attend is at an intersection inside the rectangle, and they all drive to it by their shortest possible route. Alice drives 10 blocks, Bob drives 12 blocks, and Carol drives 15 blocks. How many blocks does Don drive to get to school?

23. You have 9 silver dollars, but learn that one is likely fake. The counterfeit will look identical but weigh slightly less. Using a standard balance scale, what is the minimum number of weighings to find the bogus coin?

24. A bottle weighs 1.5 kg when it is 1/5 filled with cooking oil. It weighs 3.3 kg when it is 4/5 full. Find the weight of the empty bottle.

25. Find the area enclosed by the circle in the picture.
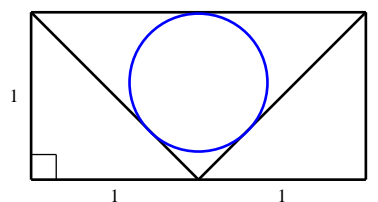


26. Let $f(x) = x^6 + 7x^3 − 8$ and $g(x) = (x^2 + x − 2)(x^2 + x + 1)$. Suppose $h(x)$ is a polynomial such that $f(x) = g(x)h(x)$. Find $h(1)$.

27. After the typist writes ten letters and addresses ten envelopes, she inserts the letters randomly into the envelopes, one letter per envelope. What is the probability that exactly nine letters were inserted in the proper envelopes?

28. An equilateral triangle and a regular hexagon have equal perimeters. What is the ratio of the area of the triangle to the area of the hexagon?

**Tie Breaker requiring Full Solution**

Please give a **detailed explanation** of your solution to **Question 18**. Write your explanation on the **reverse side** of your answer sheet. *This tie breaker question is graded as an essay question, i.e. it is graded for the clarity of explanation and argument as well as correctness.*

It is the only question graded for partial credit. Do not hesitate to write your thoughts even if your solution is not rigorous!

**It is graded only to separate first, second, and third place ties.**