2011 LSU Math Contest
Team Problems

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and fifteen minutes to complete the entire team session.

These 9 questions require exact numerical or algebraic answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact.

The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.*

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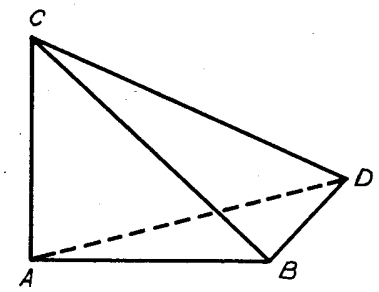
1 In the three dimensional figure



$\angle BAC = \angle DAC = \angle DBA = \angle DBC = 90°$.
Express $\tan(\angle ADC)$ in terms of trigonometric functions of the angles ABC and/or ADB and/or BDC in the figure.

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2 Given that $x = 4$ is a solution of the equation

$$a(x-1)^4 + b(x-1)^2 + c = 0,$$

find another solution of this equation.

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3 Find two real solutions x to the equation

$$4^x + 6^{x^2} = 5^x + 5^{x^2}.$$

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4 The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments of lengths 2 and 8.
Find the length of the altitude.

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5 Two circles have radii a, b. The distance between their centers is d. They intersect at P and Q.
Find the distance between P and Q.

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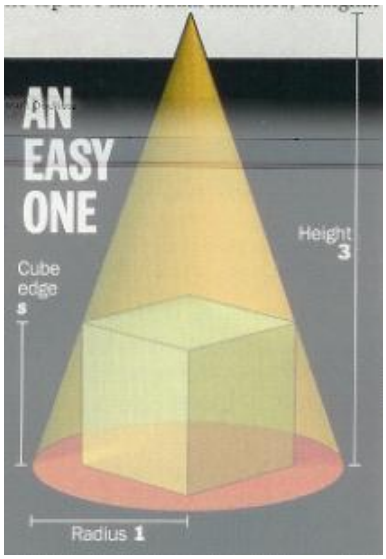
6 A right circular cone has a base of radius 1 and a height of 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the length s of an edge of the cube?



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7 At how many minutes after 8 o'clock does the minute hand lie on top of the hour hand?

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8 A man engages in a shooting contest. Each time he hits a target he receives 10 cents, while each time he misses he pays 5 cents. It, after 20 shots he has lost 10 cents, how many times has he hit the target?

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9 Let $a(n,r)$ and $s(n,r)$ stand, respectively, for $\dfrac{1-(1+r)^{-n}}{r}$ and $\dfrac{(1+r)^n - 1}{r}$.
Simplify the expression

$$\frac{1}{a(n,r)} - \frac{1}{s(n,r)}.$$
