2011 LSU Math Contest
Algebra - Geometry Session

Questions 1 - 21 are worth 1 point each and questions 22 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 21 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1
The expression $5x - (x - 6y) - 2y - (3x + 4y)$ equals

A $x - 12y$ B $9x - 4y$ C y D $10y$ E x

2
If the cost of P articles is A, the average cost per article is

A PA B A/P C P/A D $P - A$ E None of these

3
If $4ax - 3 = 15 - 2ax$, x equals

A $3/a$ B $3a$ C $4/a$ D 18 E None of these

4
Each side of an equilateral triangle is 10.
The altitude of the triangle is

A $10\sqrt{3}$ B $10/\sqrt{3}$ C $5\sqrt{3}$ D $\sqrt{10}$ E None of these

---

5
In the right triangle ABC, CD is the altitude drawn to the hypotenuse AB. If $AD = 5$, and $DB = 14$, then CD is

A $\sqrt{140}$ B 70 C $14/5$ D $\sqrt{171}$ E None of these

6
The quantity $(3x - 2a)$ is a factor of

A $9x^2 + 4a^2$

B $3x^2 - 2a^2$

C $3xy + 9bx - 2ay - 6ab$

D $9x^2 - 6ax + 4a^2$

E None of the above.

7
S varies directly with x and inversely with y^2 and $S = 9$, when $x = 12$ and $y = 2$. When $x = 18$ and $y = 3$, S equals

A 6 B 2 C 4 D 12 E None of these.

8
Two radii OA and OB of a circle intercept an arc $\overset{\frown}{AB}$ of length 1. If $OA = 2$, the area of the circular sector OAB is

A 4π B $2\pi/15$ C 2 D 1 E None of these.

9
The corresponding sides of two similar triangles are in the ratio 2 to 3. If the area of the smaller triangle is 12, the area of the larger is

A 24 B 27 C 18 D 8 E None of these.

10
The value of x in the equation $\log_x(27/8) = -3$ is

A 3 B 3/2 C 2 D 2/3 E 10

11
The complex fraction

$$(a/2 + b/3) \div (b/a + 3/2)$$

when simplified equals

A ab B $b/2$ C $ab/3$ D $ab/2$ E None of these.

12
$(x + y)^{-1} + (x - y)^{-1} =$

A $(x^2 - y^2)(2x)^{-1}$ B $(2x)^{-1}$ C $(x^2 - y^2)^{-1}$

D $2x(x^2 - y^2)^{-1}$ E None of these

13
Which of the following numbers is closest to $\frac{1}{5}\left(\frac{3}{4} + \frac{7}{8}\right)^{-1}$?

A $\frac{1}{6}$ B $\frac{1}{7}$ C $\frac{1}{8}$ D $\frac{1}{9}$ E $\frac{1}{10}$

14
If $\left(x^2 - \frac{1}{x^3}\right)^8$ is arranged as a sum of powers of x, which of the following powers of x does not appear with non-zero coefficient?

A x^{11} B x^{-4} C x^3 D x^{-19} E x

15
If 1, x, y, 27 form a geometric progression, $x =$

A 9 B 3 C $\sqrt{3}$ D $\sqrt[3]{9}$ E None of these

16
If 2, x, y, 11 form an arithmetic progression, $x =$

A 8 B 9/4 C $\frac{13}{4}$ D 5 E None of these

17
Consider the statement: "A triangle is isosceles if two of its sides are equal." In elementary geometry, this is

(A) a theorem

(B) a corollary

(C) a definition

(D) an axiom

(E) none of the above

18
A circle of radius 10 is circumscribed about a triangle ABC. If $AB = BC = 10$, then the area of the triangle is

A 50 B $25\sqrt{3}$ C $25\sqrt{2}$ D 40 E None of these

19
Which of these answers

A $x < 2$ B $x > -2$ C $x > 2$ or $x < -2$ D $-2 < x < 2$

E None of the above

satisfies the inequality

$$\frac{x^2 - 4}{x^2 + 1} > 0\ ?$$

20
At a dance, 4 girls dance with a first boy, 5 girls with a second, 6 with a third, and so on until all girls dance with the last boy. Then, if there are b boys and g girls

A $b = g + 3$ B $g = b + 4$ C $g = 4b$

D $g = b + 3$ E None of these

21
Consider a triangle with sides a, b and c such that $a^2 + b^2 = c^2$. The inscribed circle has radius r whilst the escribed (or circumscribed) circle has radius R. Find the ratio of r to R.

A $\frac{2ab}{c(a+b+c)}$ B $\frac{ab}{c(a+b+c)}$ C $\frac{2ab}{a(a+b+c)}$

D $\frac{2ab}{b(a+b+c)}$ E $\frac{2ab}{a+b+c}$

Questions 22 - 28 Exact Answers

These next seven questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ *or* 6^{40}.

---

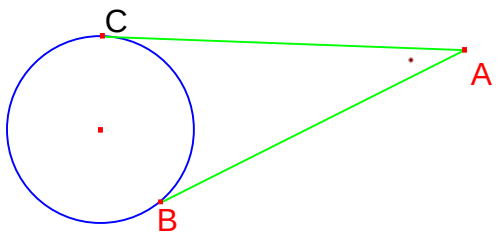
22 Find the value of $3x^0 + \left(\frac{4}{x}\right)^{-2} + \sqrt[3]{x^2}$ when $x = 8$.

---

23 A circle of radius 2 is inscribed in a regular hexagon. Find the area of the hexagon.

---

24 In the figure, lines AB and AC are tangents to the circle of radius 6 and $\angle BAC = 60°$. Find the area inside the angle $\angle BAC$ and outside the circle.



---

25 The two roots, r_1 and r_2, of the equation $x^2 + px + q = 0$ satisfy the linear equations $r_1 - 2r_2 = 2$, $2r_1 - 3r_2 = 5$. Find p.

---

26 A square of side 12 is rotated in its plane about a point on one side 3 units from a vertex. Find the area of the figure swept out.

---

27 The g.c.d. of two numbers, a and b, is D. The l.c.m. of a and b is M. Find D if $M = ab/2$.

---

28 Find all values of x for which

$$|3x - 2| + |3x + 1| = 3.$$

Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question **28**.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.

It is graded only to separate first, second, and third place ties.