2010 LSU Math Contest
Team Problems

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and fifteen minutes to complete the entire team session.

These 11 questions require exact numerical or algebraic answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact.

The tiebreaker for the team competition is time. *If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.*

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1 Determine a closed form expression for the sum

$$1 + 11 + 111 + \cdots + \underbrace{11\ldots11}_{n},$$

where the last term contains exactly n 1's.

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2 In triangle ABC, $AB = 5$ and $AC = 9$. M is the midpoint of BC, and the line through M parallel to the bisector of $\angle CAB$ cuts AC at D. Find the length of DC.

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3 If $17! = 35568x428096y00$ in decimal notation, where x and y are some digits. Find x and y.

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4 How many real solutions (x, y) does the system of equations

$$x^2 + y = 11, \quad x + y^2 = 7$$

have?

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5 If 2010^{2010} is divided by 7, what is the remainder?

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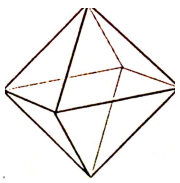
6 Find the volume of a regular octahedron all of whose vertices are on a sphere of diameter two.



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7 Find all polynomials $P(x)$ that satisfy the equation

$$P(x^2) + 2x^2 + 10x = 2xP(x + 1) + 3.$$

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8 Two circles of radius 1 intersect, the center of each being on the circumference of the other. A third circle is drawn touching the line joining the centers of the given circles, and the circumferences of both circles, the right-hand one internally and the other externally. Find the radius of the third circle.

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9 The sides of a triangle are proportional to the roots of the cubic equation

$$x^3 - ax^2 + bx - c = 0.$$

Find the sum of the cosines of the angles of the triangle in terms of a, b, and c.

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10 If in the table below we replace each letter by a number (the same letters corresponding to identical numbers) then the sums along the columns and rows are as given in the lowest row and right most column respectively. What is the missing sum of the numbers in the first row?

a	a	b	c	?
a	a	c	d	19
b	c	b	a	15
b	c	c	d	18
16	16	14	23	

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11 In a certain country the government consists of an Upper House with 6 members and a Lower House with 13 members. The Upper House is investigating the Lower House for corruption while the Lower House is investigating the Upper House for extravagance. Each of the 19 members bills the government \$600 per day up to the first 30 days in which the respective House is holding a hearing, and \$900 per day thereafter. The Upper House hearing lasts twice as many days as the Lower House hearing, and the total bill from the members of the Upper House is equal to that from the members of the Lower House. Based on this information, the Auditor General calculates and announces to the public what this fiasco may have cost the government. However, the actual figure is n times as large, where n is some number greater than 1. Determine n.
