2010 LSU Math Contest
Algebra - Geometry Session

Questions 1 - 18 are worth 1 point each and questions 19 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 18 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1

The number $\sqrt[3]{3} \cdot \sqrt[3]{5}$ is contained in the interval

A $(-\infty, 2)$ B $[2, 3)$ C $[3, 4)$ D $[4, 5)$ E $[5, \infty)$

2

Subtracting 99% of 19 from 19% of 99, the difference d satisfies

A $d < -1$ B $d = -1$ C $-1 < d < 1$ D $d = 1$ E $d > 1$

3

A difference of two different irrational numbers

A Cannot be an integer.

B Cannot be a rational number.

C Can be a rational number.

D Can be zero.

E Cannot be negative.

4

A product of three consecutive positive integers

A Is always divisible by 2 but may not be divisible by 3

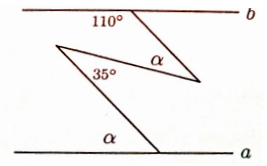
B Is always divisible by 3 but may not be divisible by 2

C May not be divisible by neither by 2 nor by 3

D Is always divisible by 6 but may not be divisible by 12

E Is always divisible by 12

5



In the diagram on the right the lines a and b are parallel.

Find the measure of angle α.

A $45°$ B $47.5°$ C $50°$ D $52.5°$ E $55°$

6

The number of solutions of $2^{3m} - 2^{2n} = 63$ in which m and n are integers is

A 0 B 1 C 2 D 3 E more than 3

7

A man spends $\frac{1}{3}$ of his money and loses $\frac{1}{3}$ of the remainder. He then has \$ 24. How much money had he at first?

A \$54 B \$108 C \$216 D \$224 E none of these

8

How many real solutions does the equation

$$\left(\frac{2x^2 - 5}{3}\right)^{x^2 - 3x} = 1$$

have?

A 0 B 2 C 4 D 6 E 8

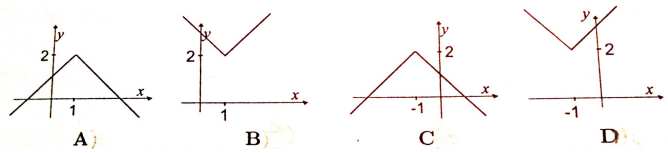
9

The vertices of an equilateral triangle of side $\sqrt{6}$ coincide with three vertices of a cube. What is the edge of the cube?

A $2\sqrt{3}$ B $\sqrt{3}$ C $\frac{\sqrt{3}}{2}$ D $\sqrt{2}$ E 1

10

Which of the following is the graph of $f(x) = 2 - |x - 1|$?



E None of the above

11
The number $\dfrac{1}{\sqrt{2}\cdot\sqrt[3]{3}}$ is equal to

A $\dfrac{\sqrt{2}\cdot\sqrt[3]{9}}{6}$ B $\dfrac{\sqrt[3]{2}\cdot\sqrt{3}}{6}$ C $\dfrac{\sqrt{2}\cdot\sqrt{9}}{6}$

D $\dfrac{\sqrt{2}\sqrt[3]{3}}{6}$ E $\dfrac{\sqrt[3]{2}\cdot\sqrt{9}}{6}$

12
Two runners race on a circular track. The first can run around the track in 6 minutes, and the second in 4 minutes. If they start off at the same point, the second runner can overtake the first in

A 12 min B 14 min C 16 min D 18 min E none of these

13
If a regular polygon has 119 diagonals,
how many sides does it have?

A 14 B 17 C 19 D 21 E 22

14
The center of a circle inscribed in a triangle is

A The point of intersection of the altitudes of the triangle.

B The point of intersection of the medians of the triangle.

C The point of intersection of the bisectors of the angles of the triangle.

D The point of intersection of the bisectors of the sides of the triangle.

E Outside of the triangle if the triangle is obtuse.

15
If $x - y < x$ and $x + y < y$, then

A $y < x$ B $0 < x < y$ C $x < y < 0$

D $x < 0,\ y < 0$ E $x < 0 < y$

16
The polynomial $P(x) = x^6 + 5x^2 - 4x - 2$

A Has at least two real roots one of which is one.

B Has at least two real roots one of which is two.

C Has only one real root.

D Has no real roots.

E None of the above is true.

17
A point P is chosen inside a square $ABCD$ such that the triangle APB is equilateral.
What is the measure of the angle DPC?

A $120°$ B $135°$ C $140°$ D $150°$ E $160°$

18
How many different lines are there that are tangent to both of two circles if the distance between the centers of the circles is greater than the sum of their radii?

A none B only one C at most two

D at most three E exactly four

Questions 19 - 28 Exact Answers

These next ten questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ *or* 6^{40}.

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19 Braille is a code which lets blind people read and write. It was invented by a blind Frenchman, Louis Braille, in 1829. Braille is based on a pattern of dots embossed on a 3 by 2 rectangle. It is read with the fingers moving across the top of the dots.
How many possible ways are there to emboss one to six dots on a 3 by 2 rectangle (assuming that we will not count zero dots)?

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20 The sum of a four-digit number and its four digits is 2010. Find all such four-digit numbers.

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21 The shadow of a 15 ft tall tree has length 20 ft. How tall is a shrub that has a shadow 4 ft long (at the same time and place) ?

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22 What are the last 5 (right-hand most) digits of $1111111111 \times 1111111111$?

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23 In the right triangle ABC: $AB = 3,\ BC = 4,\ CA = 5$. What is the area of the largest circle inside $\triangle ABC$?

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24 Find the smallest number s (expressed as a decimal) such that $|x^2 - 9| \leq s$ for all x satisfying $|x - 3| \leq 0.001$.

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25 Fill in six blanks with the digits 1 through 6, using each digit exactly once, to create the largest integer divisible by 5.

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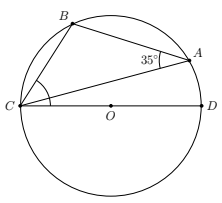
26 In the diagram on the right, O is the center of the circle, A, B, C, D are points on the circle such that the segment CD goes through O. IF $\angle BAC = 35°$, find the angle $\angle BCD$.



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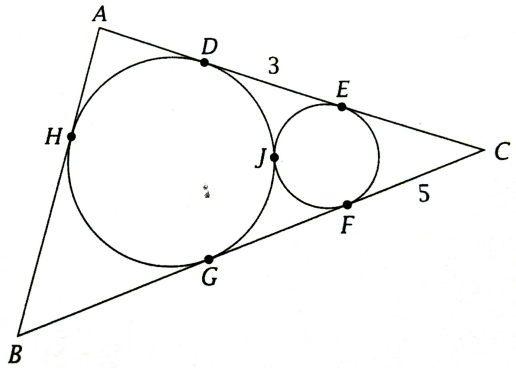
27 In triangle ABC, two circles are drawn, with tangent points at D, E, F, G, H, and J, as shown. If $DE = 3$, $CF = 5$, and the perimeter of triangle ABC is P, compute AB in terms of P.



28 A *palindrome number* is a positive integer that reads backwards the same as it reads forwards. For example: 76167.

George thought he had added together every 2-digit positive integer, and the sum he got was a palindrome number. Unfortunately, he had left one number out. What number had been omitted?

Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question **28**.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.

It is graded only to separate first, second, and third place ties.