2009 LSU Math Contest
Algebra - Geometry Session

> Questions 1 - 18 are worth 1 point each and questions 19 - 28 are worth 2 points each.
>
> No calculators are allowed.
>
> Pictures are only sketches and are not necessarily drawn to scale or proportion.
>
> You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 18 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Completely erase any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1
Which of the fractions below is the greatest?

A $\frac{7}{8}$ B $\frac{66}{77}$ C $\frac{555}{666}$ D $\frac{4444}{5555}$ E $\frac{33333}{44444}$

2
Ada has 7 gray balls, 4 white balls and 3 black balls in a bag. What is the least number of the balls she has to take out of her bag, having her eyes covered, to make sure that she took out at least one ball of each color?

A 12 B 11 C 8 D 4 E 3

3
If $a/b = 9/4$ and $b/c = 5/3$ then $(a-b)/(b-c)$ is equal to:

A 4/1 B 25/8 C 7/12 D 5/2 E Cannot be determined

4
(i) If it rains, you'll get wet.

(ii) If you get wet, you'll be sorry.

(iii) If you're not sorry, I'll be cross.

Which of the A-E is logically correct?

A You will be cross if it rains.

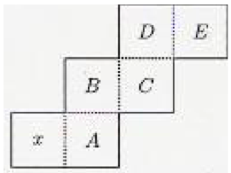
B I'll be sorry if you're cross.

C I'll be sorry if it rains.

D You'll be cross if I get wet.

E None of the above.

5
The figure shown can be folded into the shape of a cube. In the resulting cube, which of the lettered faces is opposite the face marked x ?



A B C D E

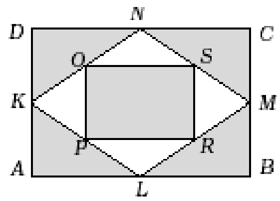
6
If $\sqrt{2+\sqrt{x}} = 3$, then $x =$

A 1 B $\sqrt{7}$ C 7 D 49 E 121

7
If $N > 1$, then $\sqrt[3]{N\sqrt[3]{N\sqrt[3]{N}}} =$

A $N^{\frac{1}{27}}$ B $N^{\frac{1}{9}}$ C $N^{\frac{1}{3}}$ D $N^{\frac{13}{27}}$ E N

8
In the diagram on the right, points K, L, M, N are the midpoints of the sides of rectangle $ABCD$ and points O, P, R, S are the midpoints of the sides of rhombus $KLMN$. What is the ratio of the area of the shaded figure to the area of rectangle $ABCD$?



A $\frac{3}{5}$ B $\frac{2}{3}$ C $\frac{5}{6}$ D $\frac{3}{4}$ E $\frac{5}{7}$

9
The sum of the lengths of the twelve edges of a closed rectangular box is 140, and the distance from one corner of the box to the farthest corner is 21. The total surface area of the box is

A 776 B 784 C 798 D 800 E 812

10
There were 25% boys and 75% girls among all the students taking part in a school event. Half of the boys and 20% of the girls, together 99 students, had blue eyes. How many students were taking part in the school event?

A 360 B 340 C 240
D Some other number E Cannot be determined

11

Suppose that for all $x > 0$ we have $f(2x) = \dfrac{2}{2+x}$.
What is $2f(x)$?

A $\dfrac{2}{1+x}$ B $\dfrac{2}{2+x}$ C $\dfrac{4}{1+x}$ D $\dfrac{4}{2+x}$ E $\dfrac{8}{4+x}$

12

Which of the following is equivalent to the algebraic fraction $\dfrac{6a+b+10}{6a+b-5}$?

A -2 B $\dfrac{15}{6a+b-5}$ C 15 D $1+\dfrac{15}{6a+b-5}$ E -1

13

The total in-store price for an appliance is \$ 99.99. A website advertises the same product for three easy payments of \$ 29.98 and a one-time shipping and handling charge of \$ 9.98. How much (in cents) is saved by buying the appliance from the website?

A 6 B 7 C 8 D 9 E 10

14

What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

A 0 B 1 C 49 D 50 E 51

15

Suppose that $P(x/3) = x^2 + x + 1$. What is the sum of all values of x for which $P(3x) = 7$?

A $-\dfrac{1}{3}$ B $-\dfrac{1}{9}$ C 0 D $\dfrac{5}{9}$ E $\dfrac{5}{3}$

A detailed explanation of the solution to this problem is the **tie breaker** *for this exam, see end of page 3.*

16

A parabola has vertex at $(4, -5)$ and has two $x-$intercepts, one positive and one negative. If this parabola is the graph of $y = ax^2 + bx + c$, which of a, b, and c must be positive?

A only a B only b C only c D a and b only E none

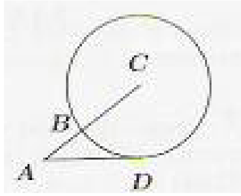
17

A speaker talked for sixty minutes to a full auditorium. Twenty percent of the audience heard the entire talk and ten percent slept through the entire talk. Half of the remainder heard one third of the talk and the other half heard two thirds of the talk. What was the average number of minutes of the talk heard by members of the audience?

A 24 B 27 C 30 D 33 E 36

18

In the diagram, C is the center of the circle and AD is tangent to the circle at D. The line $\overline{AC}$ intersects the circle at B. If $\overline{AD} = 10$ and $\overline{AB} = 7$, the radius of the circle is:



A $\dfrac{\sqrt{151}-7}{2}$ B $\sqrt{14}$ C $\dfrac{51}{14}$ D $\dfrac{\sqrt{51}}{2}$ E $\dfrac{7}{2}$

Questions 19 - 28 Exact Answers

These next ten questions require exact numerical or algebraic answers. Hand-written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized (Improper fractions can be left alone or changed to mixed fractions). Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ *or* 6^{40}.

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19 Call a 7-digit telephone number $d_1d_2d_3$-$d_4d_5d_6d_7$ *memorable* if the prefix sequence $d_1d_2d_3$ is exactly the same as either of the sequences $d_4d_5d_6$ or $d_5d_6d_7$ (possibly both). Assuming that each d_i can be any of the ten decimal digits 0, 1, 2,. . . ,9, find the number of different memorable telephone numbers.

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20 A point lies inside an equilateral triangle of side 2 and has the distances x, y, and z from the three sides respectively. Find $x + y + z$.

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21 We build a right circular cone from a cut out (white) sector of central angle of φ *radians* from a circle of radius 10 cm by taping the cut edges together.
Determine the volume of the resulting cone as a function of φ.
(Recall that the volume of the cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the cone base, h is the altitude of the cone, and 2π radians = 360°.)

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22 Find three numbers p, q, and r so that the equation

$$x^4 + 4x^3 - 2x^2 - 12x + 9 = (px^2 + qx + r)^2$$

holds identically for all real numbers x.

23 Given the equation

$$x^{2009} = y^x,$$

find all solution pairs (x, y) consisting of positive integers with x prime. (Recall that $2009 = 7 \cdot 7 \cdot 41$.)

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24 Let

$$\frac{2}{2 \cdot 5} + \frac{2}{5 \cdot 8} + \cdots + \frac{2}{2006 \cdot 2009} = \frac{x}{2009}.$$

Compute the integer x.

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25 The parabola $y = ax^2 + 19x$, where a is an integer, passes through two points in the first quadrant both of whose coordinates are integers and whose $y-$coordinates are primes. Compute the x and y coordinates of both of those points.

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26 If h hens lay e eggs in d days, how many days would it take H hens to lay E eggs?

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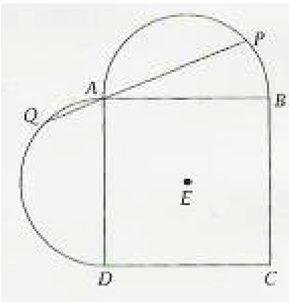
27 Semicircles are drawn on two sides of square $ABCD$, as shown. Point E is the center of the square, and QAP is a line segment with $QA = 7$ and $AP = 23$. Compute AE.



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28 In a 100-yard race, Alex beat Bob by 10 yards and Bob beat Charlie by 20 yards. Assuming that each runner ran at a constant speed, by how much did Alex beat Charlie?

Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question **15**.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.

It is graded only to separate first, second, and third place ties.