2007 LSU Math Contest
Algebra - Geometry Session

Questions 1 - 19 are worth 1 point each and questions 20 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 19 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Erase clearly any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1

Which of the following is an odd number?

A 1^4+1 B 3^4+2 C 5^4+3 D 7^4+5 E 11^4+7

2

The value of $\dfrac{3}{a+b}$ when $a=4$ and $b=-4$ is:

A 3 B $\frac{3}{8}$ C 0 D any finite number E meaningless

3

It takes 5 seconds for a clock to strike 6 o'clock beginning at 6:00 o'clock precisely. If the strikes are uniformly spaced, how long, in seconds, does it take to strike 12 o'clock?

A $9\frac{1}{5}$ B 10 C 11 D $14\frac{2}{5}$ E none of these

4

The value of $(1+\frac{1}{2})(1+\frac{1}{3})(1+\frac{1}{4})(1+\frac{1}{5})$ is:

A $\frac{1}{120}$ B $1\frac{1}{120}$ C 3 D $4\frac{1}{30}$ E $5\frac{17}{60}$

5

A box contains red and blue pencils only. If the number of red pencils is two-thirds the number of blue pencils, then the proportion of pencils in the box that are red is:

A 1:3 B 2:3 C 1:2 D 2:5 E 3:5

6

If $\dfrac{a}{d+b+c}=\dfrac{4}{3}$ and $\dfrac{a}{b+c}=\dfrac{3}{5}$, then the value of $\dfrac{d}{a}$ is:

A $\frac{7}{6}$ B $\frac{6}{7}$ C $-\frac{12}{11}$ D $-\frac{11}{12}$ E $\frac{15}{11}$

7

If $\log_{10} x - 5\log_{10} 3 = -2$, then x equals:

A 1.25 B 0.81 C 2.43 D 0.8 E either 0.8 or 1.25

8

A mixing bowl is hemispherical in shape, with a radius of 12 inches. If it contains water to half its depth, then the angle through which it must be tilted before water will begin to pour out is:

A 15° B 30° C 45° D 60° E 75°

9

A circle and a square have the same perimeter. Then:

A their areas are equal

B the area of the circle is the greater

C the area of the square is the greater

D the area of the circle is π times the area of the square

E none of these

10

The negation of the statement "all men are honest," is:

A no men are honest

B all men are dishonest

C some men are dishonest

D no men are dishonest

E some men are honest

11

The expression

$$\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$$

can be written as

A $\dfrac{3+\sqrt{6}+\sqrt{15}}{6}$ B $\dfrac{\sqrt{6}-2+\sqrt{10}}{6}$ C $\dfrac{2+\sqrt{6}+\sqrt{10}}{10}$

D $\dfrac{2+\sqrt{6}-\sqrt{10}}{6}$ E none of these

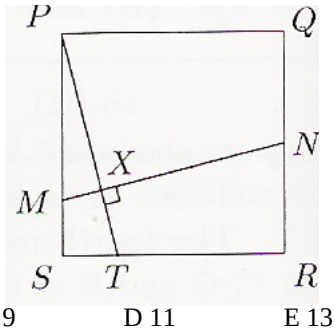
12

A fruit stand buys mangoes at 3 for \$ 1. They will sell them at 5 for \$ 2. In order to make a profit of \$ 10.00, they must sell:

A 67 mangoes B 150 mangoes C 200 mangoes
D an infinite number of mangoes E none of these

13

$PQRS$ is a square of side 12. T is a point on RS such that $ST = 5$. MN is perpendicular to PT and intersects PT at X. If $MX = 4$ then the length of XN is



A 5 B 7 C 9 D 11 E 13

14

Circle C_1 passes through the center of, and is tangent to, circle C_2. The area of circle C_1 is 4. Then the area of circle C_2 is:

A 8 B $8\sqrt{2}$ C $8\sqrt{\pi}$ D 16 E $16\sqrt{2}$

15

The price of an article was increased p%. Later the new price was decreased p%. If the last price was one dollar, the original price, in dollars, was:

A $\dfrac{1-p^2}{200}$ B $\dfrac{\sqrt{1-p^2}}{100}$ C 1 D $1-\dfrac{p^2}{10,000-p^2}$ E $\dfrac{10,000}{10,000-p^2}$

16

Which of the following is true about the equation

$$x^3 - 1 = (x-1)\left(x^2 - x + 1\right) ?$$

A It has exactly one real solution

B It has exactly two real solutions

C It has exactly three real solutions

D It has infinitely many solutions, but it is not an identity

E It is true for any value of x

17

Given that $0 < a < b < c < d$, which of the following is the largest?

A $\dfrac{a+b}{c+d}$ B $\dfrac{a+d}{b+c}$ C $\dfrac{b+c}{a+d}$ D $\dfrac{b+d}{a+c}$ E $\dfrac{c+d}{a+b}$

18

Let $x = 0.7181818\ldots$, where the digits '18' repeat. When x is expressed as a fraction in lowest terms, then its denominator exceeds its numerator by:

A 18 B 31 C 93 D 141 E 279

19

Six numbers from a list of nine integers are 7, 8, 3, 5, 9, and 5. What is the largest possible value of the median of all nine numbers in this list?

A 5 B 6 C 7 D 8 E 9

Questions 20 - 28 Exact Answers

These next nine questions require exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ *or* 6^{40}.

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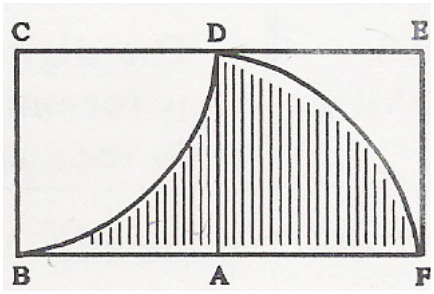
20 $ABCD$ and $ADEF$ are squares with a common side AD of length 12 inches. Arc BD and arc DF are quarter-circles.
How many square inches are in the area of the shaded region?



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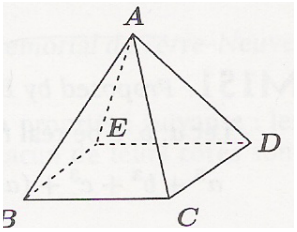
21 The pyramid $ABCDE$ has a square base, and all four triangular faces are equilateral.
Find the measure of the angle BAD (in degrees).



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22 When the stem of a water lily is vertical, the blossom is 10 centimeters above the surface of the lake. Pulling the lily to one side, keeping the stem straight, the blossom touches the water at a point 21 centimeters from where the stem formerly cut the surface. How deep is the water in the lake?

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23 If four mice can eat four pounds of cheese in four minutes, how long will it take 99 mice to eat 99 pounds of cheese?

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24 In a fraction, its denominator exceeds its numerator by 3521. The fraction expressed in lowest terms is $\frac{4}{11}$. What was the original fraction before simplification?

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25 Find the length of the diameter of a circle whose area is tripled when the length of its radius is increased by 2.

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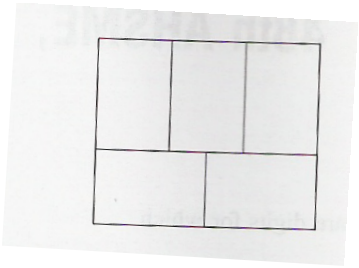
26 A rectangle with perimeter 176 is divided into five congruent rectangles as shown in the diagram. What is the perimeter of one of the five congruent rectangles?



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27 The table shows the distance (in km) between five towns in southern France. Five friends live in these five towns and want to meet. In which town should they meet to keep the total travelling distance as small as possible?

Bergerac				
87	Bordeaux			
79	47	Langon		
61	31	54	Libourne	
58	84	37	65	Marmande

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28 Some tarts had been eaten without Mrs Karen's permission, by one or more of her five children. When questioned, they gave the following answers: -

Ace: One of us ate the tarts.

Bea: Two of us ate the tarts.

Cec: Three of us ate the tarts.

Dee: Four of us ate the tarts.

Eve: All of us ate the tarts.

Mrs Karen knew from the past behavior of her children that the guilty ones lied while the others told the truth. What was the number of children who ate the tarts?

Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question **28**.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.

It is graded only to separate first, second, and third place ties.