2006 LSU Math Contest
Algebra - Geometry Session

Questions 1 - 19 are worth 1 point each and questions 20 - 28 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 19 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Erase clearly any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1

The value of $[2 - 3(2 - 3)^{-1}]^{-1}$ is

A 5 B -5 C $\frac{1}{5}$ D $-\frac{1}{5}$ E $\frac{5}{3}$

2

The greatest value of $1 - 2\cos\theta$ is

A 5 B 3 C 1 D 0 E -1

3

On a calculator $\frac{1}{3} = 0.3333333$. What would $\frac{1}{30}$ be?

A 3.3333333 B 0.3030303 C 0.3333333

D 0.0303030 E 0.0333333

4

$\frac{8}{5}$ equals

A 0.625 B 1.667 C 1.8 D 1.6 E 0.6

5

A cube has a side length of 4 in. The cube is painted red all over and then cut into 64 cubes of side 1 in. How many of these cubes have exactly one face painted red?

A 6 B 16 C 24 D 36 E 64

6

The function $f(x) = -|x|$ satisfies the following condition for all real numbers x and y

A $f(x + y) = f(x) + f(y)$ B $f(x - y) = f(x) - f(y)$

C $f(x \cdot y) = f(x)f(y)$ D $f(-x \cdot y) = -f(x)f(y)$

E none of the above

7

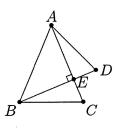Four consecutive odd integers are added. If the smallest one is $2m - 1$, then the sum equals

A $8m - 10$ B $8m + 2$ C $8m + 8$

D $8m + 10$ E $8m + 3$

8

Triangles ABC and ABD are isosceles with $AB = AC = BD$, and $\overline{BD}$ intersects $\overline{AC}$ at E. If $\overline{BD} \perp \overline{AC}$, then $\angle C + \angle D$ is

A $115°$ B $120°$ C $130°$ D $135°$ E not uniquely determined

9

The flag is given a half-turn counterclockwise about the point O and is then reflected in the dotted line.

Which picture shows the correct final position of the flag?

10

A number is divided by 6 and then 3 is subtracted from the result to give 4. The original number is

A 60 B 27 C 45 D 54 E 42

11

If the numbers denoted below are written in the order of size, which will be in the middle?

A $\frac{1}{3}$ B $\frac{3}{10}$ C 31% D 0.03 E 0.303

12

Which of these numbers is the average (mean) of the other four?
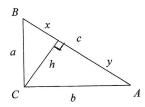
A 28 B 30 C 26 D 37 E 29

13

Consider a right triangle ABC with hypotenuse $AB = c$

and the following proposed formulas for the height h of the $\triangle ABC$:

(a) $h = \sqrt{xy}$ (b) $h = \frac{ab}{c}$ (c) $h = \frac{a^2+b^2}{xy}$

Which of these formulas are true?

A only (a)   B only (b)   C only (c)

    D (a) and (b)   E (b) and (c)

14

The numbers 1, 3, 6, 10, 15,... are known as *triangular numbers*. Each triangular number can be expressed as $\frac{n(n+1)}{2}$ where n is a natural number. The largest triangular number less than 500 is:

A 494   B 495   C 496   D 497   E 498

15

$2^n - 2^{n-2} = 192$. What is the value of n?

A 5   B 6   C 7   D 8   E 9

16

In a regular hexagon the diagonals increased by 30%. The area of the hexagon increased then by about

A 30%   B 40%   C 50%   D 60%   E 70%

17

A car travels one mile in one minute. To travel one mile in 40 seconds the car must increase its speed by

A 20%   B 30%   C 40%   D 50%   E 60%

18

The equation

$$| - x| = -x$$

has

A no solutions   B one solution   C two solutions

    D three solutions   E more than three solutions

19

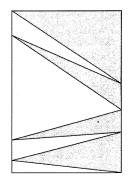The last (ones) digit of a perfect square cannot be:
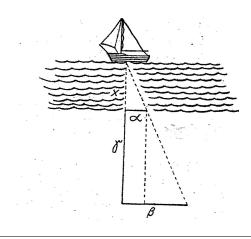
A 1   B 4   C 5   D 6   E 8

Questions 20 - 28 Exact Answer Questions

These next nine questions are exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out $20!$ or 6^{40}.

20 What fraction of the area of the rectangle below is its shaded part?

21 Write the following three numbers in a nondecreasing order: $\sqrt{2}$, $\sqrt[3]{3}$, and $\sqrt[4]{4}$.

22 An ancient Greek philosopher, scientist and mathematician, Thales of Miletus, found a distance from a ship to the seashore using a triangle and a trapezoid which he drew with his cane on the beach. Repeat his computation and find x in terms of the sides $\alpha, \beta,$ and γ of the trapezoid as marked on the picture below.
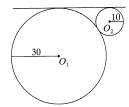

23 Paul has twice as many brothers as sisters. His sister Mary has five times as many brothers as sisters. How many sons and daughters do their parents have?

24 Find a function $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(-x) = -f(x) \quad \text{and} \quad f(x+1) = f(x) + 1$$
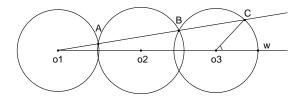
for all $x \in \mathbb{R}$.

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25 A diamond weighing 4 carats broke into two pieces: a 1-carat and a 3-carat. Express the loss of the value of the diamond as a fraction of its original value if a diamond's value is directly proportional to the square of it's weigh.

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26 Two circles with centers O_1 and O_2 and radii of length 30 and 10 respectively are tangent to each other and to a line as shown.

Find the area of the region between the two circles and the line.

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27 One of the legs of a right triangle has length 15 in. The radius of the inscribed circle has length 3in. Find the area of the triangle.

28 All the circles below are the same size with centers $o1$, $o2$, $o3$ and with upper points of intersection A, B respectively from left to right ; $o1ABC$ and $o1o2o3W$ are straight lines.

Find the ratio of measure of the angle $\angle Co3W$ to the measure of the angle $\angle Co1W$.

Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question **28**.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.

It is graded only to separate first, second, and third place ties.