2005 LSU Math Contest
Team Problems

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and fifteen minutes to complete the entire team session.

These 11 questions (except for question 4) require exact numerical or algebraic answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact.

The tiebreaker for the team competition is time. If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.

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1 A googol is a 1 followed by 100 zeros (in base 10) or 10^{100} or 10 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 . The term was coined by a nine year old asked by his father to come up with a name for this big number. A much larger number is a googolplex, which in base 10 is a 1 followed by a googol zeros. It is too big to write down in digits–even using all the paper in the world. Perhaps we could write it down if we chose a different base, which is very large. If you were to write a googolplex in base googol, then it would be a 1 followed by how many zeros?

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2 A grocer weighted four fruits: a lemon, a pear, an apple, and a grapefruit. The following observations were made:

(a) the lemon weighted more than the pear,

(b) the combined weight of the pear and the apple was greater than the combined weight of the lemon and the grapefruit, and

(c) the combined weight of the lemon and the pear was equal to the combined weight of the apple and the grapefruit.

List the four fruits in order of weight from heaviest to lightest.

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3 Find the surface area of a cube which just fits in a sphere of radius 1 in.

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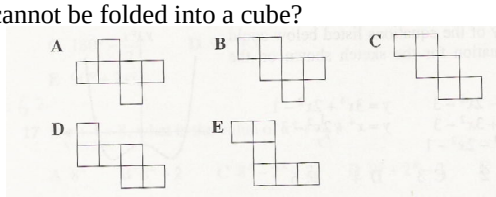
4 Which of the following configurations of six squares cannot be folded into a cube?



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5 Some shepherds lose two-thirds of their flock and then find four-fifths of these. What fraction of the flock did they have left?

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6 Let α be the angle between two faces of a regular tetrahedron. Find $\cos\alpha$.

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7 Find all positive integers n for which the equation

$$|x| + |x+1| + |x+2| + \cdots + |x+n| = n$$

has an integer solution x.

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8 There are two times between 3PM and 4 PM when the hands of an accurate clock are perpendicular. Exactly how many minutes must elapse between these two times?

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9 A pentagon is formed by cutting a triangular corner from a rectangular piece of paper. The five sides of the pentagon have lengths 13, 19, 20, 25, and 31, although this is not their order around the pentagon. What is the area of the pentagon?

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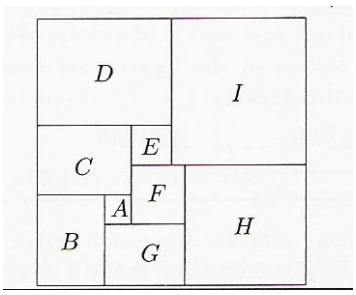
10 Nine labeled squares are arranged as shown.



If square A has area 1 in^2 and square B has area 81 in^2 then what is the area, in square inches, of square I ?

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11 A two-headed coin, a two-tailed coin and an ordinary coin are placed in a bag. One of the coins is drawn at random and flipped; it comes up "heads." What is the probability that there is a head on the other side of this coin?