2005 LSU Math Contest
Open Session

Questions 1 - 13 are worth 1 point each and questions 14 - 23 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

1. Questions 1 - 13 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Erase clearly any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1

$\dfrac{1}{ab+b^2}+\dfrac{1}{a^2+ab}$ equals

A $\dfrac{1}{ab}$ B $\dfrac{1}{a^2+b^2}$ C $\dfrac{a^2+b^2}{ab}$ D $\dfrac{a+b}{ab}$ E $\dfrac{2}{a^2+2ab+b^2}$

2

A new operation $*$ for combining two numbers a and b is defined by

$$a*b := \frac{a+b}{2}.$$

If $x*(x*4)=x$, what is x ?

A 0 B 1 C 2 D 3 E 4

3

The expression $\frac{k}{3}(k+1)(k+2)+(k+1)(k+2)$ is equal to

A $\frac{1}{6}(k+1)(k+3)(k+4)$ B $\frac{1}{3}k(k+1)(k+2)$ C $\frac{1}{3}(k+1)(k+2)(k+3)$ D $\frac{2k}{3}(k+1)(k+2)$ E $\frac{1}{4}(k+1)(2k+1)(3k+2)$

4

If $9^{3-x}=81^{1-2x}$ then x equals

A -2 B $\frac{2}{7}$ C $-\frac{1}{3}$ D $\frac{8}{7}$ E 4

5

A polyhedron with twelve edges cannot be a

A cube B octahedron C pyramid D dodecahedron E none of the above

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6

The equation

$$|2^x-2| = -2x^2+4x-2$$

has

A no solutions B one solution C two solutions D three solutions E four solutions

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7

Let p, q be positive integers with $p>q$. Then 2^{p-q} is equal to

A $\sqrt[q]{2^p}\sqrt[p]{2^q}$ B $(\sqrt[p+q]{2})^{p^2-q^2}$ C $2^{p+q}2^{2p}$ D $2^{p+q}/2^{2p}$ E none of the above

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8

If $x=\sec\theta+\tan\theta$ and $y=\sec\theta-\tan\theta$ then which of the following are true for all values of θ for which x and y are defined?

A $\frac{x+y}{2}=1$ B $xy=1$ C $x^2+y^2=2$ D $x^2+y^2=1$ E $\frac{x-y}{2}=1$

9

Consider a cone of height 2. The area of a cross section of the cone with a plane parallel to its base is 25% of the area of the base.
The distance between the base and the plane cannot be greater than

A $\frac{1}{4}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{3}{4}$ E 1

10

There exist positive integers m, n, and p with greatest common factor 1, such that

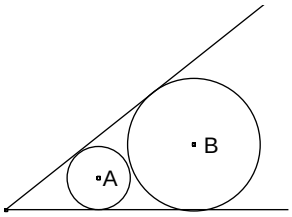
$$m\log_{200}5+n\log_{200}2=p\,.$$

What is $m+n+p$?

A 6 B 7 C 8 D 9 E 10

11

A circle with center A and radius 7 is tangent to the sides of an angle of 60°. A larger circle with center B is tangent to the sides of the angle and to the first circle.



The radius of the larger circle is

A $30\sqrt{3}$ B 21 C $20\sqrt{3}$ D 30

E not enough information to determine the radius

12

In a tetrahedron, all sides are of length 1 except possibly one whose length is denoted by x. Let $V(x)$ be the volume of the tetrahedron. Which of the following statements is true ?

A $V(x)$ is an increasing function which has no maximum

B $V(x)$ is an increasing function which has a maximum

C $V(x)$ is not an increasing function and has no maximum

D $V(x)$ is not an increasing function but has a maximum

E none of the above

13

On a multiple choice test, one of the test questions was missing, however the answer choices were

A All of the below

B None of the below

C All of the above

D Exactly one of the above

E None of the above

Which answer is correct?

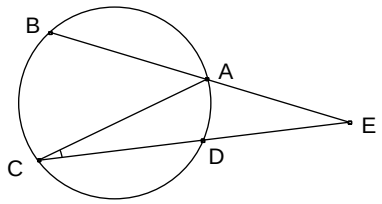
2. Questions 14 - 23 Exact Answer Questions

These next ten questions are exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out 20! *or* 6^{40}.

14 Given that $2\sqrt{2-\sqrt{3}}=\sqrt{a}-\sqrt{b}$, where a and b are positive integers, find a and b.

15 What is the domain of the function $f(x)=\ln(x^3-x)$?

16 In the circle below the arcs AB, BC, and CD are equal and less than half circle. Given that $\angle CEB=40^\circ$ find $\angle DCA$ (in degrees).



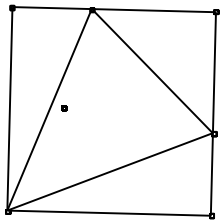
17 The sequence $6,-9,x,y$ is such that the first three terms form an arithmetic sequence and the last three terms form a geometric sequence. Find the values of x and y.

18 If $\cos A=\frac{3}{5}$ and $\pi<A<2\pi$, find $\tan A$.

19 Find all real solutions to the equation

$$5^{|1-4x^2|} = \sin(\pi x)\,.$$

20 A square of unit area contains an equilateral triangle as shown. Calculate the length of the sides of the equilateral triangle.



21 Let x and y be real numbers. Find all pairs (x,y) whose sum $x+y$, product $x\cdot y$, and quotient $\frac{x}{y}$ are all equal, or state that no such pair exists.

22 A square is partitioned into four congruent smaller squares by lines parallel to its sides. A non-zero digit has been written in each smaller square. This way we get four double digit numbers: two when you read the square horizontally and two when you read it vertically. The sum of those four double digit numbers is 67. What are all the four digits inscribed in the four smaller squares?

23 You are on a game show on television. On this game show the idea is to win a car as a prize. The game show host shows you three doors. He says that there is a car behind one of the doors and there are goats behind the other two doors. He asks you to pick a door. You pick a door but the door is not opened. Then the game show host opens one of the doors you didn't pick to show a goat (because he knows what is behind the doors). Then he says that you have one final chance to change your mind before the doors are opened and you get a car or a goat. So he asks you if you want to change your mind and pick the other unopened door instead.
What is the probability of you winning the car if indeed you change your mind and pick up the other door?

3. Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question **23**.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.
It is graded only to separate first, second, and third place ties.