2005 LSU Math Contest
Algebra - Geometry Session

Questions 1 - 15 are worth 1 point each and questions 16 - 24 are worth 2 points each.

No calculators are allowed.

Pictures are only sketches and are not necessarily drawn to scale or proportion.

You have one hour and twenty minutes to complete the entire morning exam.

Questions 1 - 15 Multiple Choice

Please:
- *Use the answer sheet for your answers.*
- *Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.*
- *Erase clearly any answer you wish to change.*
- *Do not make stray marks on the answer sheet.*

1

Let $x = 2 + \sqrt{3}$ and $y = 2 - \sqrt{3}$. Then the quotient $\frac{x}{y}$ is

A an irrational number B 1 C $7 + \sqrt{4}$

D 66 2/3 E none of these

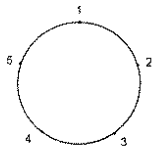
2

The height of an equilateral triangle of side a is given by

A $\frac{a\sqrt{2}}{2}$ B $\frac{a\sqrt{3}}{2}$ C $\frac{a\sqrt{3}}{4}$ D $\frac{a\sqrt{2}}{3}$ E $\frac{a\sqrt{2}}{4}$

3

Five points on a circle are numbered 1, 2, 3, 4, and 5 in clockwise order. A bug jumps in a counterclockwise direction from one point to another around the circle. If it is on an odd-numbered point, it moves two points, and if it is on an even-numbered point, it moves one point. If the bug begins on point 1, after 2005 jumps it will be on point



A 1 B 2 C 3 D 4 E 5

4

The value of $1-2+3-4+5-6+\cdots-998+999-1000+1001$ is

A 500 B 501 C -501 D -1001 E 1000

5

The sizes of the angles of a quadrilateral are x, $x+10$, $x+20$, and $x+30$ degrees.

The largest angle, in degrees, is

A 75 B 85 C 95 D 105 E 115

6

If a linear function $f(x)$ satisfies the following two conditions

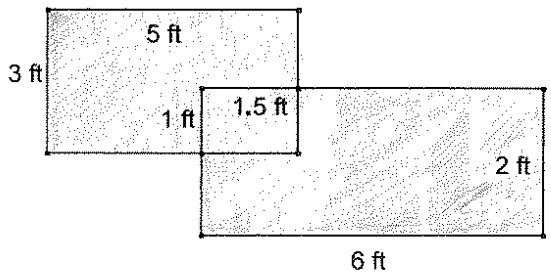
$$1)\ f(1) = 3, \quad 2)\ f(x) = f(x+1) - 2,$$

then $f(x) =$

A $2x+1$ B $x+2$ C $2x$ D $4x-1$ E $2.5x+.5$

7

Two rectangles overlap in a smaller rectangle as shown.

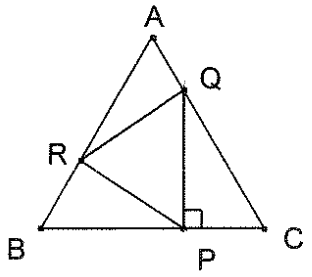


Find the shaded area in square feet.

A 25.5 B 27 C 24 D 26.5 E 28.5

8

Equilateral triangle PQR is inscribed in equilateral triangle ABC as shown below with $PQ \perp BC$.

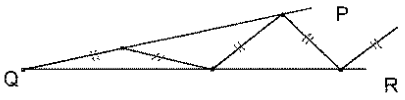


The ratio of the area of $\triangle PQR$ to the area of $\triangle ABC$ is

A $\frac{1}{6}$ B $\frac{1}{4}$ C $\frac{1}{3}$ D $\frac{2}{5}$ E $\frac{1}{2}$

9

In the diagram $\angle PQR = 12°$, and a sequence of isosceles triangles is drawn as shown.



What is the largest number of such triangles that can be drawn?

A 8 B 4 C 5 D 7 E 6

10

How many positive integers less than 1000 have the sum of their digits equal to 6?

A 18 B 19 C 27 D 28 E 111

11

How many times in one (24 hours) day do the hands of a clock form a right angle?

A 22 B 24 C 44 D 46 E 48

12

Apples and pears used to cost the same. One day the price of apples went up 6% and the price of pears 10%.

The price of 4 lb of apples and 4 lb of pears together will have increased

A 6% B 8% C 10% D 16% E none of these

13

Mr. Smith was driving 50 m.p.h. for the first two hours of his trip, and then 60 m.p.h. for the remaining three hours of his trip.

What was his average speed (in m.p.h.) during the whole trip?

A 54 B 55 C 56 D 57 E 58

14

If $x^2 + \frac{1}{x^2} = 62$, what is the value of $x^4 + \frac{1}{x^4}$?

A 8^4 B $8^4 + 2$ C $8^4 - 2^8 + 2$

D $8^4 + 2^8 - 2$ E $8^4 + 2^8$

15

$1/a + 1/b + 1/c$ equals

A $\frac{3}{a+b+c}$ B $\frac{a+b+c}{abc}$ C $\frac{3(a+b+c)}{abc}$

D $\frac{ab+ac+bc}{abc}$ E $\frac{3}{abc}$

Questions 16 - 24 Exact Answer Questions

These next nine questions are exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, i.e., do not try to multiply out 20! or 6^{40}.

16 Vita Veggie Restaurant sells dumplings in boxes of four, seven, and thirteen. Find the largest number of dumplings that you cannot buy in a combination of Vita's boxes.

17 Given that $2\sqrt{2-\sqrt{3}} = \sqrt{a} - \sqrt{b}$, where a and b are positive integers, find a and b.

18 For a real number x let $[x]$ denote the greatest integer $\leq x$, e.g. $[4.99] = 4$. Compute x if $[x] \cdot x = 11$.

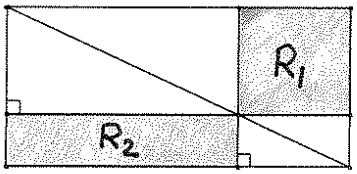
19 The roots x_1 and x_2 of the quadratic equation $x^2 + ax = b - 1$ are non-zero integers. Write $a^2 + b^2$ as a product of two integers greater than one.

20 Find the exact value of

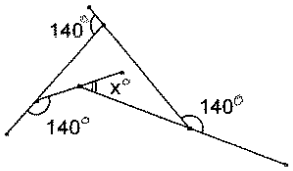
$$(666\,666\,666)^2 - (333\,333\,333)^2.$$

21 What is 30% of 40% of 50?

22 The area of the shaded rectangle R_1 is 1 in^2. Find the area of the shaded rectangle R_2



23 What is the size of the angle x in the diagram below?



24 Consider the equation $2x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 3$.

How many non-negative integral solutions does it have?

Tie Breaker

Please give a detailed explanation on the answer sheet to your solution to Question **24**.

This tie breaker question is graded as an essay question i.e., it is graded for the clarity of explanation and argument as well as correctness. It is the only question graded for partial credit.

It is graded only to separate first, second, and third place ties.