Open Session

Questions 1-13 are worth 1 point each and questions 14 -23 are worth 2 points each.

Questions 1-13 Multiple Choice

Complete instructions are on a separate page, but please:

- Use the answer sheet for your answers.
- Answer only one choice A, B, C, D, or E for each question by circling your answer on the answer sheet.
- Erase clearly any answer you wish to change.
- Do not make stray marks on the answer sheet.

1

In the number 0.1234512345... (recurring) what is the 2004th digit after the decimal point?

A 1 B 2 C 3 D 4 E 5

2

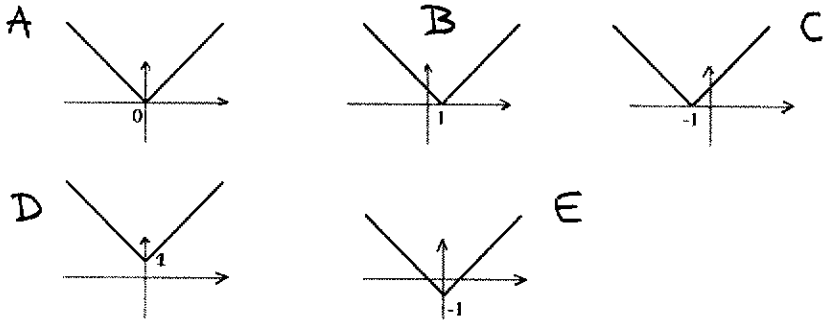
Factor $x^2 + 9x + 20$.

A $(x+6)(x+3)$ B $(x+2)(x+10)$ C $(x-1)(x-20)$

D $(x+4)(x+5)$ E none of these

3

Which of the following pictures best approximates the graph of $y = |x-1|$?



4

What is the domain in the real numbers of the real-valued function $f(x) = \ln(x^3 - x)$?

A $x > 0$ B $x > 1$ C $x < 0$ D $x < -1$ E none of these

5

What is the value of $\cos(\sin^{-1} 1)$?

A -1 B -1/2 C 0 D 1/2 E 1

6

The diameter of one circle equals the radius of a second circle. Find the ratio of their areas.

A 1:2 B 1:3 C 1:4 D 1:5 E 1:8

7

Let m and n be two positive integers. Consider the following two statements about m and n:

(I) The number $m - n$ is odd.
(II) The number $m^2 - n^2$ is odd.

Which of the following statements is true?

A (I) is necessary but not sufficient for (II)
B (I) is sufficient but not necessary for (II)
C (I) is neither necessary nor sufficient for (II)
D (I) is necessary and sufficient for (II)
E can't be sure

8

If $\sin x + \cos x = 1.1$, then the value of $\sin x \cos x$ is

A 1.1 B 0.105 C 0 D 0.21 E 1.05

9

The three positive real numbers x, y, z satisfy the system of two equations:

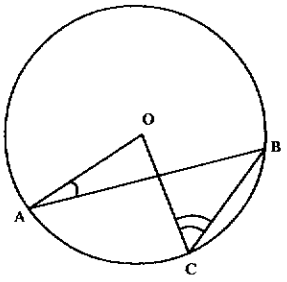
$$\frac{z}{x+y} = 2, \qquad \frac{z}{x-y} = 3.$$

Which of the following is true:

A $x < y < z$ B $y < z < x$ C $z < x < y$ D $y < x < z$ E $x < z < y$

10

In the diagram, O is the center of the circle, $\angle OAB = 10°$ and $\angle OCB = 30°$. Then $\angle ABC =$



A 10° B 20° C 30° D 40° E 50°

11

Two dice are rolled, each with two black, two green and two red faces. What is the probability that both dice show matching colors?

A 1/216 B 1/108 C 1/36 D 1/12 E 1/3

12

If one root of the equation $x^3 - 5x^2 + 5x - 1 = 0$ is $2 - \sqrt{3}$, then the sum of the other two roots is

A $-7 + \sqrt{3}$ B $-1 + \sqrt{3}$ C $3 + \sqrt{3}$ D $-3 + \sqrt{3}$ E 5

13

The following 'proof' that $1 = 2$ of course contains a mathematical error. Which line (A, B, C, D or E) is not a consequence of the previous one?

Let $a = b$

A $ab = b^2$

B $ab - a^2 = b^2 - a^2$

C $a(b - a) = (b + a)(b - a)$

D $a = b + a$

E when $a = 1$ and $b = 1$, we get $1 = 1 + 1 = 2$.

Questions 14-23 Exact Answer Questions

These next ten questions are exact numerical or algebraic answers. Hand written exact answers must be written on the answer sheet with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, *i.e.*, do not try to multiply out 20! or 6^{40}.

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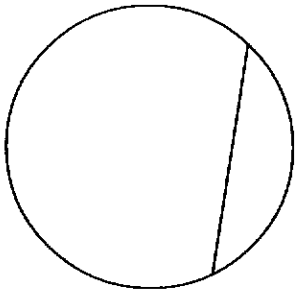
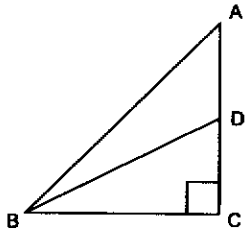
14. A parabola with vertical axis of symmetry has vertex (0, 8) and has an x-intercept of 2. Find the equation of the parabola.

15. How many real roots at most can the equation $x|x| + px + q = 0$ have?

16. If the figure below is a circle of radius 1 unit, and the chord has a length of one unit, what is the area of the smaller region between the chord and the circle?



17. ABC is an isosceles right triangle with right angle at C, and D is the midpoint of AC. Find $\tan(\angle ABD)$.



18. If you have 2 one dollar bills, 2 quarters and 3 nickels, how many different but non zero sums can you pay without making change?

19 A janitor has 8 master keys at home to open all the rooms in a school. Each room can be opened by just one of these keys. If 40% of the rooms are left unlocked, what is the probability that the janitor can get into a specific room if he selects three keys at random before leaving home to go to work?

20. Let $y = \log_{\frac{1}{\sqrt{2}}} \frac{1}{x+3}$. Determine the values of x for which $y < 0$.

21. Solve the equation $\log_x(19x - 30) = 3$.

22 Find all polynomials $P(x)$ satisfying for all real x the equation

$$x^2P(x) + P(1 - x) = 2x - x^4.$$

23. If one and a half painters can paint one and a half walls in one and a half days, how many walls can six painters paint in nine days?

Name:_____

School:_____

Open Session: Tiebreaker

This last page is the tiebreaker question. This question is graded as an essay question *i.e.*, it is graded for the clarity of explanation and argument as well as correctness. It is graded only to separate first, second, and third place ties. It is the only question graded for partial credit. You may use the back or this page if you need more space.

Question

Consider the table

$$\begin{aligned} 1 &= 1 \\ 3+5 &= 8 \\ 7+9+11 &= 27 \\ 13+15+17+19 &= 64 \\ 21+23+25+27+29 &= 125 \end{aligned}$$

Guess the general law suggested by these examples, express it in suitable mathematical notation, and prove it.
