Open Session

Questions 1-16 are worth 1 point each and questions 17-25 are worth 2 points each.

Questions 1-16 multiple choice

Complete instructions are on a separate page, but please:
• Answer only one choice a, b, c, d, or e for each question.
• Only use a number 2 pencil.
• Make a heavy black mark that fills the cirlce.
• Erase clearly any answer you wish to change.
• Do not make stray marks on the answer sheet.

1. The line defined by $3y + 7x = 4$ has slope

a. 7   b. -7   c. -7/3   d. -3/7   e. None of the choices

2. The graph of the relation $\dfrac{(y-1)^2}{4} = 1 + \dfrac{(x-1)^2}{4}$ is

a. a circle   b. an ellipse   c. a hyperbola   d. a parabola   e. a line

3. How many different solutions are there to the equation

$$|x + |3x - 2|| = 2.$$

a. 0   b. 1   c. 2   d. 3   e. 4

4. Suppose that θ is a number and $0 < \theta < 90$. Consider the following statements and select the correct choice.

1. $\sec(\theta°) < 1$
2. $\sec(\theta°) = 1$
3. $\sec(\theta°) > 1$

a. only 1 is possible   b. only 3 is possible   c. only 1 is impossible
d. only 3 is impossible   e. all are possible

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5. Which of the following pictures best approximates the graph of $y = (x-1)^3(x+1)^2$?

(a)　　　　　(b)　　　　　(c)

(d)　　　　　(e)
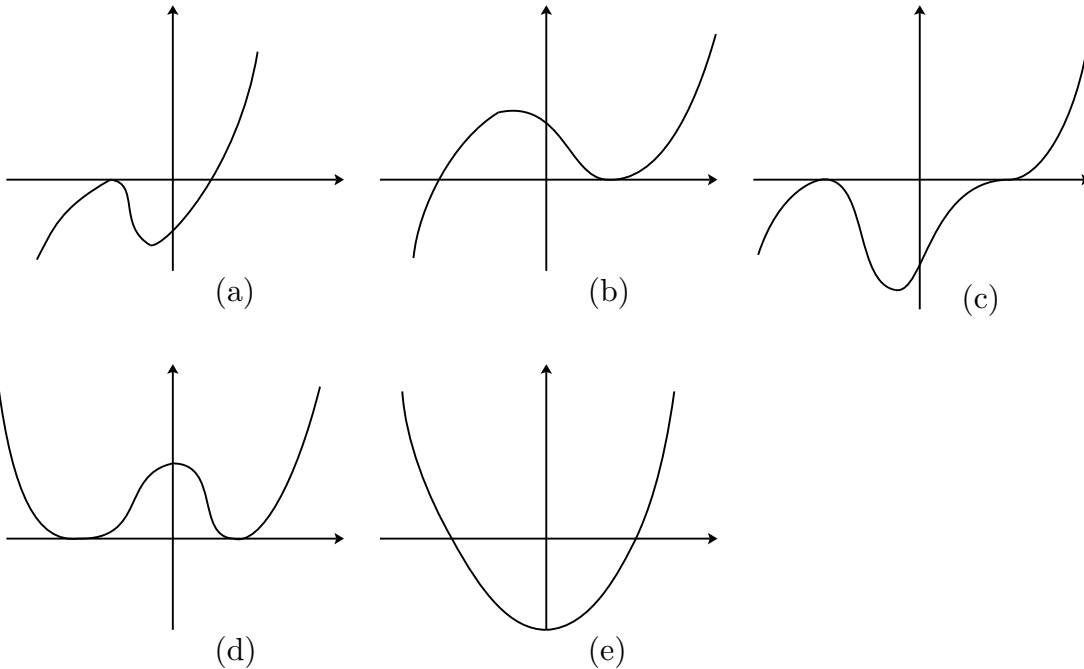
6. The measure of an angle is twice the measure of its supplement. What is the degree measure of the angle?

a. 90　　b. 120　　c. 60　　d. 30　　e. None of the choices

7. Which of the following is true about base 10 logarithms?

a. $\log(x+1) = \log x + \log 1$ for all $x > 0$.
b. $(\log x)(\log 1) = 0$ for all $x > 0$.
c. $\frac{\log(x+2)}{\log(x-1)} = \log(x+2) - \log(x-1)$ for all $x > 1$.
d. $10^x = \frac{1}{\log x}$
e. More than one of the above choices are true

8. The polynomial

$$p(x) = x^7 - 6x^6 - 12x^5 + 200x^4 - 720x^3 + 1248x^2 - 1088x + 384$$

has 2 as a root of multiplicity 6, i.e., it occurs six times. Find another root of $p(x)$.
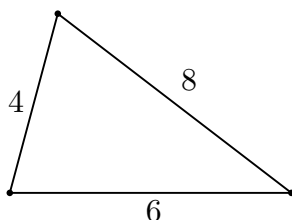
a. 4　　b. -4　　c. -3　　d. 6　　e. None of the choices

9. From 1900 to 1950 the population of East Baton Rouge Parish increased 4920% and from 1950 to 2000 the population increased 150%. What was the average percentage increase per half century?

a. $(4920 + 150)/2$ b. $100(1 + \frac{4920}{100})(1 + \frac{150}{100})/2 - 100$ c. $(\frac{4920}{100} + \frac{150}{100})/2$

d. $100\sqrt{(1 + \frac{4920}{100})(1 + \frac{150}{100})} - 100$ e. None of the choices

10. The formula for the area of a triangle in terms of the length of its sides is called Hero's formula, but it was first discovered by Archimedes. If the side lengths of a triangle are a, b, c, then let s denote the semiperimeter $\frac{1}{2}(a + b + c)$. The area of the triangle is then $[s(s - a)(s - b)(s - c)]^{\frac{1}{2}}$. Using the information above, find the area of the triangle below.

a. $9\sqrt{15}$ b. 24   c. 12   d. $6\sqrt{3}$ e. None of the choices

11. If the operation $\diamond$ is defined as $a \diamond b = a^{b-1}$ for $a, b > 0$, then $3 \diamond (2 \diamond 3)$ is

a. 27   b. 9   c. 3   d. 1   e. None of the choices

12. The next question concerns the new Louisiana tax rates for joint filers. The income listed in the chart is state taxable income. There were other tax changes not relevant to this question.

TaxRate	2%	4%	6%
Old	Up to $20,000	$20,000 to $100,000	Over $100,000
New	Up to $25,000	$25,000 to $50,000	Over $50,000

The tax rates above are marginal tax rates. That means if you earn 40,000 dollars, then you pay 2% tax on 25,000 dollars and 4% tax on 40,000-25,000=15,000 dollars. If a couple had state taxable income of $100,000 in both last year and this year, then by what percentage will their taxes increase? Answer rounded to the nearest whole number.

a. 1   b. 5   c. 10   d. 25   e. None of the choices

13. Euclid's definition of a point is

a. A point is that which is breadthless and depthless.
b. A point is that which is the least of all line segments.
c. A point is that which is the meeting of two lines.
d. A point is that which has no part.
e. A point is that which is dimensionless.

14. The equation $\dfrac{1}{16}2^{x^2} = 2^{3x}$ has

a. no solutions   b. one solution   c. two solutions
d. three solutions   e. more than three solutions

15. The equation $\dfrac{2}{x+2} + \dfrac{1}{x-3} = \dfrac{5}{x^2-x-6}$ has

a. no solutions   b. one solution   c. two solutions
d. three solutions   e. more than three solutions

16. For which real values of a does the system

$$ax + y = 6$$
$$4x - ay = -2$$

have a unique solution?

a. Only for $a \neq 2$ b. Only for $a \neq \pm 4$ c. Only for $a < 2$
d. Only for $a \neq \pm 2$ e. None of the choices

## Questions 17-25 Exact Answer Questions

These questions require exact numerical answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, *i.e.*, do not try to multiply out 20! or 6^{40}.

17. If $\tan(\alpha^\circ) = \frac{1}{3}$ where $180 < \alpha < 270$ and $\cos(\beta^\circ) = \frac{1}{3}$ where $270 < \beta < 360$, then compute $\sin((\alpha - \beta)^\circ)$.

18. Compute the sum

$$\log_2 |\cos(24^\circ)| + \log_2 |\cos(48^\circ)| + \log_2 |\cos(96^\circ)| + \log_2 |\cos(192^\circ)|$$

19. In the triangle $\triangle ABC$, the measure of $\angle A$ is twice the measure of $\angle C$. Also the length of $\overline{AB}$ is 2 and the length of $\overline{BC}$ is 3. Find the length of $\overline{AC}$.
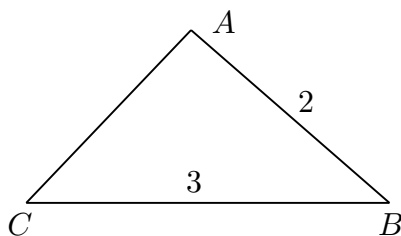
20. The set of points (x, y) whose distance from the line $y = 2x + 2$ is the same as the distance from (x, y) to the point $(2, 0)$ is a parabola. This parabola is congruent to a parabola in standard form $y = kx^2$ for some number k. Find k.
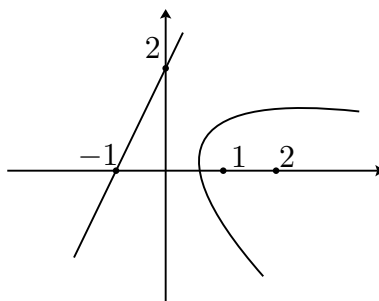

21. Suppose X is the 100 digit base 10 number that is represented by ninety nine 9's followed by a 7, i.e., $X = 9 \cdots 97$. What is the remainder in the division problem

$$10^{300} \div X?$$

22. Suppose A and B are points on the circle with center C. The angle $\angle ACB$ is 30°. If a point D is randomly chosen on the circle, then what is the probability that the triangle $\triangle ABD$ is obtuse?

23. Recall that the area of an ellipse defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab. The intersection of the cylinder $\{(x, y, z) \mid x^2 + y^2 = 1\}$ and the plane $\{(x, y, z) \mid z = 2y\}$ is an ellipse. Find the area of the ellipse.

24. Suppose $\triangle ABC$ is an equilateral triangle and D is an interior point such that $\overline{AD} = 3$, $\overline{BD} = 4$ and $\overline{DC} = 5$. Find the degree measure of $\angle ADB$.
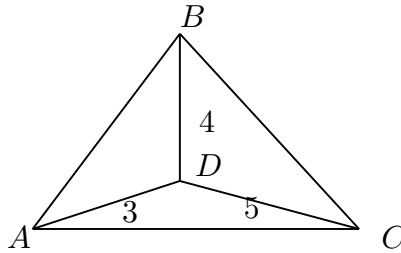
25. Three people take turns drawing a card from a shuffled deck, returning the card and reshuffling after each draw. They agree that whoever draws a heart first will pay for dinner. If in the first set of drawings no one draws a heart, they draw again, taking turns in the same order, and they continue in this manner until eventually someone draws a heart. What is the probability that the third person will have to pay for dinner?

Name:

School:

Open Session: Tiebreaker

This last page is the tiebreaker question. This question is graded as an essay question *i.e.*, it is graded for the clarity of explanation and argument as well as correctness. It is graded only to separate first, second, and third place ties. It is the only question graded for partial credit. You may use the back or this page if you need more space.

If x is a whole number and the tens place digit of x^2 is 5, then what are all of the possible tens place digits of x.