Algebra-Geometry Session

Questions 1-16 are worth 1 point each and questions 17-25 are worth 2 points each.

Questions 1-16 multiple choice

Complete instructions are on a separate page, but please:
- Answer only one choice a, b, c, d, or e for each question.
- Only use a number 2 pencil.
- Make a heavy black mark that fills the cirlce.
- Erase clearly any answer you wish to change.
- Do not make stray marks on the answer sheet.

1. The line defined by $3y + 7x = 4$ has slope

a. 7 b. -7 c. -7/3 d. -3/7 e. None of the choices

2. The sum of the degree measures of the interior angles in a quadrilateral is

a. 180 b. 360 c. 540 d. 720 e. None of the choices

3. Each Sunday a newspaper agency sells x copies of the newspaper for \$1 each. The agency buys the newspapers for 40 cents each. Also the agency pays a fixed cost of \$100 for delivery fees. Which of the following is an expression for the profit in dollars in terms of the number of newspapers the agency sells.

a. $1.4x + 100$ b. $\frac{6}{10}x - 100$ c. $1.4x - 100$ d. $60x - 100$ e. None of the choices

4. If $x = 3$ and $y = -5$, then $(x + y)x + y$ is

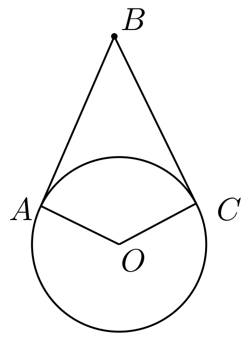
a. 4 b. -4 c. -11 d. -1 e. None of the choices

5. The measure of an angle is twice the measure of its supplement. What is the degree measure of the angle?

a. 90 b. 120 c. 60 d. 30 e. None of the choices

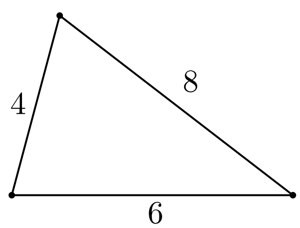
6. In the diagram below, O is the center of the circle, $\overline{AB}$ and $\overline{BC}$ are tangent lines to the circle with points of tangency A and C respectively. If $\angle O$ measures $90°$ then the degree measure of $\angle B$ is

a. 90 b. 120 c. 60 d. 45 e. None of the choices



7. The formula for the area of a triangle in terms of the length of its sides is called Hero's formula, but it was first discovered by Archimedes. If the side lengths of a triangle are a, b, c, then let s denote the semiperimeter $\frac{1}{2}(a+b+c)$. The area of the triangle is then $[s(s-a)(s-b)(s-c)]^{\frac{1}{2}}$. Using the information above, find the area of the triangle shown.

a. $9\sqrt{15}$ b. 24 c. 12 d. $6\sqrt{3}$ e. None of the choices



8. From 1900 to 1950 the population of East Baton Rouge Parish increased 4920% and from 1950 to 2000 the population increased 150%. What was the percentage increase during the 20th century?

a. $4920 + 150$ b. $(4920)(150)$ c. $(1 + \frac{4920}{100})(1 + \frac{150}{100}) - 1$
d. $100(1 + \frac{4920}{100})(1 + \frac{150}{100}) - 100$ e. $(100 + 4920)(100 + 150) - 100$

9. This problem concerns the same information as question 8. Recall that from 1900 to 1950 the population of East Baton Rouge Parish increased 4920% and from 1950 to 2000 the population increased 150%. What was the average percentage increase per half century?

a. $(4920 + 150)/2$ b. $100(1 + \frac{4920}{100})(1 + \frac{150}{100})/2 - 100$ c. $(\frac{4920}{100} + \frac{150}{100})/2$
d. $100\sqrt{(1 + \frac{4920}{100})(1 + \frac{150}{100})} - 100$ e. None of the choices

10. A math club from Greensboro, AL took a bus trip to the Greensboro Open Math Contest in Greensboro, NC. The city of Atlanta is 40% of the way from Greensboro AL to Greensboro NC. If the bus averaged 60 mph from Greensboro AL to Atlanta and 50 mph from Atlanta to Greensboro NC, then what was the average speed of the bus for the whole trip?

a. $\dfrac{1}{\frac{0.4}{60} + \frac{0.6}{50}} \approx 53.6$
b. $(0.4)60 + (0.6)50 = 54$
c. $\frac{50+60}{2} = 55$
d. $(0.4)50 + (0.6)60 = 56$
e. It cannot be determined without more information: either a distance or a length of time.

11. Let $f(n) = n/((n-1)/\cdots/(3/(2/1))\cdots)$. Which of the following is true?

a. $f(7) = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$ b. $f(7) = \frac{7 \cdot 5 \cdot 3}{6 \cdot 4 \cdot 2}$ c. $f(7) = \frac{6 \cdot 4 \cdot 2}{7 \cdot 5 \cdot 3}$
d. $f(7) = 1$ e. None of the choices

12. If the operation $\diamond$ is defined as $a \diamond b = a^{b-1}$ for $a, b > 0$, then $3 \diamond (2 \diamond 3)$ is

a. 27 b. 9 c. 3 d. 1 e. None of the choices

13. How many different solutions are there to the equation $|x + |3x - 2|| = 2$.

a. 0 b. 1 c. 2 d. 3 e. 4

14. Euclid's definition of a point is

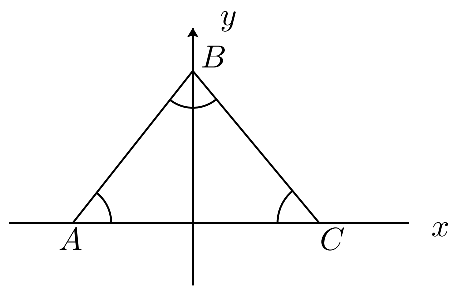
a. A point is that which is breadthless and depthless.
b. A point is that which is the least of all line segments.
c. A point is that which is the meeting of two lines.
d. A point is that which has no part.
e. A point is that which is dimensionless.

15. The equation $\dfrac{2}{x+2} + \dfrac{1}{x-3} = \dfrac{5}{x^2 - x - 6}$ has

a. no solutions b. one solution c. two solutions
d. three solutions e. more than three solutions

16. Suppose the triangle $\triangle ABC$ shown in the diagram is an equilateral triangle. If the line $\overline{BC}$ is described by an equation of the form $y = mx + b$ for some m and b, then find m.

a. 60 b. 90 c. -1/2 d. $-\sqrt{3}$ e. -60



Questions 17-25 Exact Answer Questions

These questions require exact numerical answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact. Large numbers should not be multiplied out, *i.e.*, do not try to multiply out 20! or 6^{40}.

17. The polynomial

$$p(x) = x^7 - 6x^6 - 12x^5 + 200x^4 - 720x^3 + 1248x^2 - 1088x + 384$$

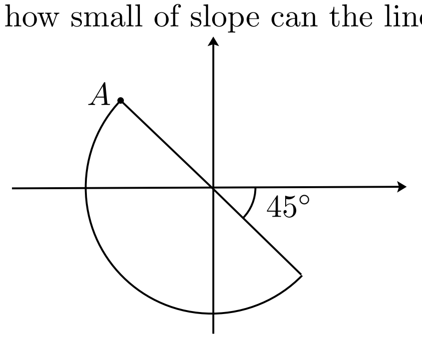
has 2 as a root of multiplicity 6, i.e., it occurs six times. Find another root of $p(x)$.

18. In the year 2070 the International Collegiate Athletic Association (ICAA) replaced the college basketball season with a large 1000 team single elimination tournament. In a single elimination tournament a team that wins a game in a round advances to the next round and a team that loses is eliminated from the tournament. Once a team plays, they play in every subsequent round until eliminated. The best teams from the previous year start in the second round, they get a bye or get to bypass the first round. All other teams start in the first round. How many teams get a bye in the 1000 team basketball tournament?

19. Suppose X is the 150 digit base 10 number that is represented by one hundred fourty nine 9's followed by a 5, i.e., $X = 9\cdots95$. What is the remainder in the division problem

$$10^{300} \div X?$$

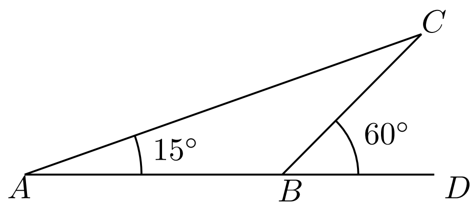
20. A line is called a supporting line of a figure if the line meets the figure but the whole figure is on one side of the line. For example, the line that contains the side of a square is a supporting line of the square. Consider the figure at the right drawn on the coordinate plane—it is half of the unit circle . If a supporting line passes through A, then how large of slope can the line have and how small of slope can the line have?



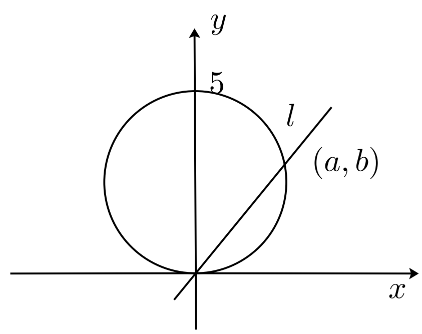
21. Suppose A and B are points on the circle with center C. The angle $\angle ACB$ is 30°. If a point D is randomly chosen on the circle, then what is the probability that the triangle $\triangle ABD$ is obtuse?

22. If x is a whole number and the tens place digit of x^2 is 5, then what are all of the possible ones place digits of x.

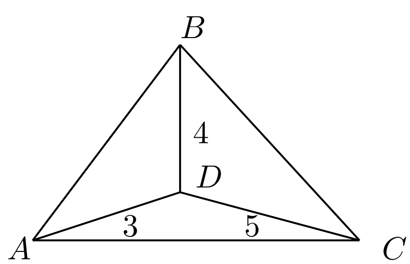
23. In the diagram below the measure of $\angle CAB$ is $15°$ and of $\angle CBD$ is $60°$. If the length of $\overline{BC}$ is 1, then find the length of $\overline{AB}$.



24. Suppose the origin, $(0, 5)$, and (a, b) are on a circle whose diameter is on the y-axis. Let l be the line that passes through $(0, 0)$ and (a, b). If $a^2 + b^2 = 16$ and $a > 0$, then find the slope of l.



25. Suppose $\triangle ABC$ is an equilateral triangle and D is an interior point such that $\overline{AD} = 3$, $\overline{BD} = 4$ and $\overline{DC} = 5$. Find the degree measure of $\angle ADB$.



Name:____

School:____

Algebra-Geometry Session: Tiebreaker

This last page is the tiebreaker question. This question is graded as an essay question *i.e.*, it is graded for the clarity of explanation and argument as well as correctness. It is graded only to separate first, second, and third place ties. It is the only question graded for partial credit. You may use the back or this page if you need more space.

Question: For what values of m does the equation $16x^2 + 16mx + 4m - 1 = 0$ have an integer solution for x?