## Team Problems

These questions require exact numerical answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact.

The tiebreaker for the team competition is time. If your team reaches a point where you are satisfied or expect that you will not have more solutions in the allotted time, then you may wish to turn in your paper a little early to get a time advantage.

1. A consultant unpacks six laptop computers, uses them to conduct a training session, and then repackages them for shipment back to her company. After repackaging the last computer she notices that each box has on it a serial number that should be the same as the number on the machine within. The repackaging unfortunately was conducted randomly. What is the probability that at least one of the boxes contains the correct computer?

2. Mary estimated that she needed to plow 50 acres a day in order to be ready for spring planting. After plowing exactly half the land at that average rate, she realized that she should have been plowing at a rate of 60 acres per day to meet the deadline. How many acres each day will she need to average in order to meet the deadline?

3. The diagram below is in the complex plane. The circle has radius 2 and center 0. If the angle $\angle ABC$ measures 30^0, then find the complex number A. Express your answer in the form $a + bi$.
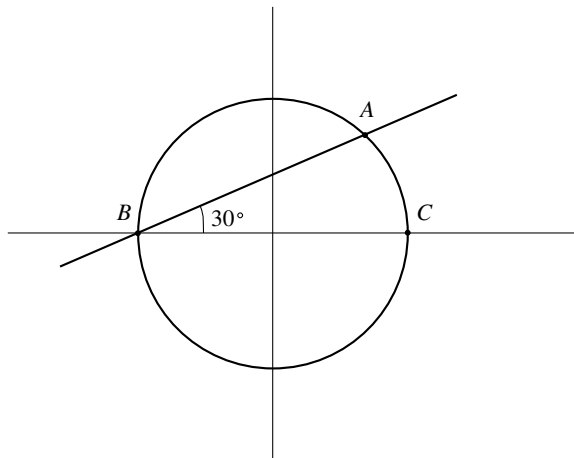
4. Consider the triangle below with its inscribed circle. The length of the sides of the triangle are 1, 2, and $\sqrt{3}$. Points A, B, and C are points of tangency. Find the length of the small arc AC.

2   A

B   1

C

$\sqrt{3}$
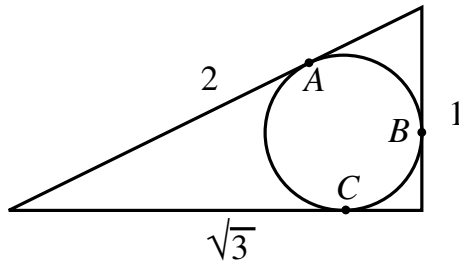
5. Let $f(x) = \frac{x}{x-1}$. Compute,

$$f(f(f(f(f(f(f(3 + \sqrt{3})))))))).$$

6. If the number $7^{(3^{2002})}$ is written as an ordinary base 10 numeral, then what is the digit in the ones place.

7. In the triangle below, α is an angle with $\sin \alpha = \frac{2}{7}$. The arc is a portion of the circle of radius 1 and center at A. The arc is tangent to $\overline{BC}$ at X. Find the length of BC.

B

X

1

α

A   C
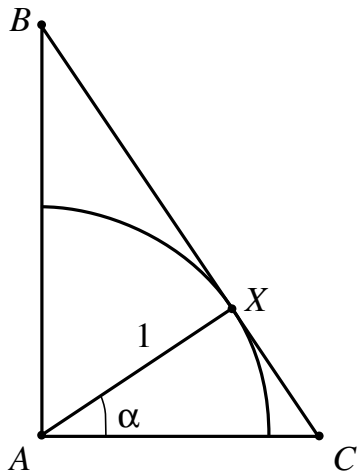
8. A triangle is inscribed in a circle of diameter 2 so that one side of the triangle is a diameter. If one of the remaining sides of the triangle subtends an arc with angular measure 30^0, find the area of the circle this triangle circumscribes.
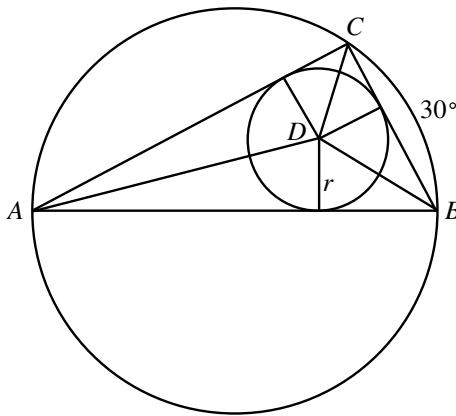
9. On 03/9/02, Charles amused himself with the following computation. Multiplying his age (in years) by 12 and adding to that the product of 25 and a certain three digit number, he obtained a result of 3464, which happened to be the last four digits of his phone number. To his surprise, he discovered two other three digit integers that allowed him to repeat the computation using first his father's age and then his grandfather's age, each time obtaining 3464. How old was Charles' grandfather on 03/9/02.

10. The three-digit number 2a3 is added to the number 326 to give the three-digit number 5b9. If 5b9 is divisible by 9, find a + b.

11. On an auto trip, the distance read from the instrument panel was 450 miles. With snow tires on for the return trip over the same route, the reading was 440 miles. Find the increase in radius of the wheels if the original radius was 15 inches.

12. Find the number of digits in the number $N = 2^{12} \times 5^8$.

13. After finding the average of 35 scores, an absent-minded professor carelessly included the average with the 35 scores and found the average of these 36 numbers. What was the ratio of the second average to the true average?