Open Session Problems
Questions 1-20 multiple choice

- Answer only one choice: (a), (b), (c), (d), or (e) for each of the following questions.
- Only use a number 2 pencil.
- Make heavy black marks that fill the circle.
- Erase clearly any answer you wish to change.
- Do not make stray marks on the answer sheet.

1. The roots of $(x^2 - 3x + 2)(x)(x - 4) = 0$ are:

   (a) 4
   (b) 0 and 4
   (c) 1 and 2
   (d) 0,1,2 and 4
   (e) 1,2 and 4

2. Which of the following points is on an asymptote of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1?$$

   (a) (16,9)
   (b) (12,16)
   (c) (9,4)
   (d) (9,16)
   (e) None of the above.

3. The slope of the line $x + 3y - 24 = 0$ is

   (a) $\frac{1}{3}$
   (b) 3
   (c) -3
   (d) $-\frac{1}{3}$
   (e) None of the above

4. Solve for x in the equation $\frac{1}{x} + \frac{3}{2x} = 6$.

   (a) $\frac{5}{12}$
   (b) $\frac{2}{9}$
   (c) $\frac{1}{10}$
   (d) $\frac{12}{5}$
   (e) None of the above.

5. If $\sec(30^0) = x$, then x is

   (a) $\frac{\sqrt{3}}{2}$
   (b) $\frac{1}{2}$
   (c) 2
   (d) $\frac{2\sqrt{3}}{3}$
   (e) $\sqrt{3}$

6. The vertex of the parabola defined by $x^2 + 4x + 2y + 10 = 0$ is located at

   (a) (2,3)
   (b) (-3,2)
   (c) (-2,-3)
   (d) (2,5)
   (e) (5,2)

7. $\sqrt{\pi}\sqrt[3]{\pi} =$

   (a) π^{-5}
   (b) π^5
   (c) $\pi^{\frac{1}{5}}$
   (d) $\pi^{\frac{5}{6}}$
   (e) None of the above.

8. The system

$$2x + 5y = 10 \quad (1)$$
$$4x + 4 = -10y \quad (2)$$

   (a) has one solution $(\frac{1}{5}, \frac{1}{2})$
   (b) has one solution $(\frac{1}{5}, -\frac{1}{2})$
   (c) has one solution $(-5, 2)$
   (d) has no solutions
   (e) has infinitely many solutions.

9. Three times the measure of an angle is the measure of its supplement. What is the degree measure of the angle?

   (a) 30
   (b) 45
   (c) $\frac{45}{2}$
   (d) 60
   (e) None of the above.

10. Find cos(arctan(2)).

   (a) $\frac{1}{2}$
   (b) $\frac{1}{3}$
   (c) $\frac{\sqrt{2}}{2}$
   (d) $\frac{\sqrt{3}}{3}$
   (e) None of the above.

11. Which of the three following definitions are functions of x? Take a number x between 0 and 90, draw a right triangle ABC with angle A measuring x degrees and call the opposite side a.

   Let $f(x)$ be the length of the hypotenuse,
   Let $g(x)$ be the length of the hypotenuse measured in inches and
   Let $h(x)$ be the length of the hypotenuse divided by the length of side a.

   (a) All of f, g and h are functions.
   (b) Only f and g are functions.
   (c) Only g is a function.
   (d) Only h is a function.
   (e) None of the above is correct.

12. Which of the following statements are true?

(I): $-1 < x < 2$ implies $0 < x^2 < 4$

(II): $-1 < x < 2$ implies $1 < x^2 < 4$

(III): $-1 < x < 2$ implies $-1 < x^2 < 4$

(a) Only I.

(b) Only II.

(c) Only III.

(d) Both I and II.

(e) All of I, II, and III.

13. How **many** solutions are there to the equation

$$\cos x^\circ = (\frac{x}{360})^2?$$

(a) no solutions

(b) 1 solution

(c) 2 solutions

(d) 3 solutions

(e) more than 3 solutions

14. Solve for x in the equation

$$2^{(2^x)} = 16^{(16^{16})}.$$

(a) 62

(b) 63

(c) 64

(d) 65

(e) None of the above

15. If m people can do a job in d days, then m+r people can do the job in:

(a) d+r days

(b) d-r days

(c) md/(m+r) days

(d) d/(m+r) days

(e) None of the above

16. The second of two numbers is 5 less than twice the first. Three times the first, decreased by 4 times the second, is 10. Find the two numbers.

    (a) -6 and -7
    (b) $\frac{-10}{11}$ and $\frac{-35}{11}$
    (c) $\frac{30}{11}$ and $\frac{5}{11}$
    (d) 2 and 1
    (e) None of the above

17. The number 64 is divided into three parts with proportionality 2:4:6. The smallest part is:

    (a) $5\frac{1}{3}$
    (b) 11
    (c) $10\frac{2}{3}$
    (d) 5
    (e) None of the above

18. Completely factor the polynomial $x^3 - 2x^2 - 2x$.

    (a) $x(x^2 - 2x - 2)$
    (b) $x(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})$
    (c) $x(x + 1 + \sqrt{3})(x + 1 - \sqrt{3})$
    (d) $x(x - 2)(x + 1)$
    (e) None of the above is correct.

19. If you were to expand $(x + y)^{10}$ to a polynomial with 11 terms, then what is the sum of the coefficients?

    (a) 0
    (b) 102
    (c) 1024
    (d) 2052
    (e) None of the above.

20. An equation of the vertical line through $(\pi, \sqrt{2})$ is

    (a) $y = \sqrt{2}$
    (b) $y = \pi$
    (c) $x = \pi$
    (d) $x = \sqrt{2}$
    (e) None of the above

Exact Answer Questions

These next ten questions require exact numerical answers. Hand written exact answers must be written with fractions reduced, radicals simplified, and denominators rationalized. Do not make an approximation for π or other irrational numbers. Answers must be exact.

1. Simplify the following expression and then evaluate it at $x = 0$

$$\csc^6(x) - 3\csc^6(x)\cos^2(2x) + 3\csc^6(x)\cos^4(2x) - \csc^6(x)\cos^6(2x).$$

2. Suppose r is a root of the polynomial $x^4 - x^3 + x^2 - x + 1$. What is $r^{20} - r^{15} + r^{10} - r^5 + 1$?

3. A cube is 100 inches on each side. The cube is painted blue and then cut into 1,000,000 small cubes with 1 inch sides. How many of the small cubes have exactly one side painted blue?

4. A parallelogram has perimeter 40 and altitudes of length 4 and 7. Find the sine of the smaller vertex angle.

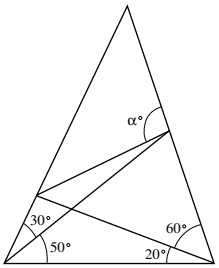
5. A sequence $a_1, a_2, a_3, \cdots$ is the increasing sequence of the numbers that are sums of distinct nonzero powers of 3:

$$3, 9, 12, 27, 30, 36, 39, \cdots$$

   What entry is $6561 = 3^8$, i.e., if $a_n = 6561$, then find n.

6. A right circular cone of base radius 1 and slant height 3 is given. P is a point on the circumference of the base. Find the length of the shortest path from P around the cone and back to P.

7. Consider the triangle below. Find the angle α in degrees.



8. What is the numerical value of

$$\log_2(3)\log_3(4)\log_4(5)\log_5(6)\log_6(7)\log_7(8).$$

9. The circle with center at $(0, 1)$ and radius 1 intersects the parabola defined by $y = ax^2$ at $(0, 0)$. What is the largest a so that the circle and the parabola intersect at $(0, 0)$ and nowhere else?

10. How many right triangles with sides having whole number lengths have perimeter less than 100?

Tie Breaker Questions

This last page is the tiebreaker question. You should give an argument. It is an essay question, *i.e.*, it is graded for its clarity as well as its correctness. It is graded only to separate first, second and third place ties. It is the only question graded for partial credit.

Question: Find all solutions to

$$x + y + z = xyz$$

where x, y, z are integers.