Spring Mini Courses in Analysis and Geometry

Fuglede's Conjecture

February 8 - 11, 2018

Speaker: Palle E.T. Jorgensen - *University of Iowa*

The lectures and discussion-seminars on the Fuglede Conjecture will focus on three themes:

(1) Vector fields and spectral theory: History of the problem: paper by Fuglede, connections to I.E. Segal and to J. von Neumann. Unbounded Hermitian operators, selfadjoint extensions, boundary values and spectral theory. Some non-commutative analogues.

(2) The two sides of the Fuglede-conjecture/problem: A closer look at tiles vs orthogonal Fourier frequencies (spectral pairs). Tao et al, dimensions 3 and higher, vs dimension 1 and 2. The case of d-cubes. The universal tiling conjecture.

(3) Fractals and representation theory: Fractal variants of the conjecture. Scaling, self-similarity, fractal limits, wavelets on fractals, Fourier series on affine fractals, spectral pairs revisited, fractals in the large. Ergodic theoretic constructions. Representations of the Cuntz relations.

References

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