Verify the following results using some version of induction. Write your arguments out completely, being sure to identify the statement $P(n)$ appropriately (or the subset S of the positive integers that you will be showing is all of the positive integers).

1. Show that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for all integers $n \geq 1$.

2. Show that $2^{2n-1} + 1$ is divisible by 3 for all $n \geq 1$.

3. Show that $f_2 + f_4 + \cdots + f_{2n} = f_{2n+1} - 1$ for all $n \geq 1$, where f_n denotes the n^{th} Fibonacci number.

4. Show that for all $n \geq 1$,
$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n} = 2 - \frac{n+2}{2^n}.$$

5. Show that $2^n < n!$ for all $n \geq 4$. Recall that for a positive integer n, $n! = n(n-1)(n-2)\cdots 2 \cdot 1$.

6. Show that any integer $n \geq 12$ can be written as a sum $4r + 5s$ for some nonnegative integers r, s. (This problem is sometimes called a postage stamp problem. It says that any postage greater than 11 cents can be formed using 4 cent and 5 cent stamps.)