Instructions. Answer each of the questions on your own paper, and be sure to show your work, including giving reasons, so that partial credit can be adequately assessed. Put your name on each page of your paper. There is a total possible of 100 points.

1. [**18 Points**]
   (a) Find the greatest common divisor $d = (4307,\, 1121)$ using the Euclidean Algorithm, and write $d = (4307,\, 1121)$ in the form $d = 4307 \cdot s + 1121 \cdot t$.
   (b) Find integers x and y such that $4307x + 1121y = 236$ or explain why it is not possible.
   (c) Compute the least common multiple $m = [4307,\, 1121]$.
2. [**10 Points**] Prove that $(3a + 5,\, 2a + 3) = 1$ for any integer a.
3. [**16 Points**] Let $a = 2^3 5^8 13^2$, $b = 2^5 3^7 5^3$. Find the following:
   (a) The prime factorization of $(a,\, b)$.
   (b) The prime factorization of $[a,\, b]$.
   (c) The largest e such that $2^e \mid a^3 b$.
   (d) The largest f such that $5^f \mid (a + b)$.
4. [**18 Points**] Let p and q denote distinct primes and recall that
   $\tau(n) =$ the number of positive divisors of n $\qquad \sigma(n) =$ the sum of the positive divisors of n.
   (a) Find the prime factorization of 350.
   (b) $\tau(350) =$
   (c) $\sigma(350) =$
   (d) $\tau(p^4 q^2) =$
   (e) Give the positive divisors of $p^4 q^2$ (table form is fine).
   (f) What are the prime factorizations possible for n if $\tau(n) = 10$.
5. [**12 Points**] Use the definition of divisibility to prove that if $a \mid b$ and $a \mid (c - 3b)$ then $a \mid c$.
6. [**12 Points**] Prove that $2^{n+2} \mid (2n + 3)!$ for all positive integers n.
   (Recall that $m! = \prod_{j=1}^{m} j = m(m-1)(m-2)\cdots 2 \cdot 1$.)
7. [**14 Points**] Circle True (T) or False (F). Reasons are not required for this problem.

T	F	$a \mid 0$ for all non-zero integers a.
T	F	If $a \nmid b$ and $a \nmid c$ then $a \nmid bc$.
T	F	If $a \mid d$ and $b \mid d$ then $(a,\, b) \mid d$.
T	F	If $d \mid a$ and $d \mid b$ then $d \mid (a,\, b)$.
T	F	Any integer can be written as a linear combination of 12 and 15.
T	F	$496 = 2^4 \cdot 31$ is a perfect number.
T	F	If $\tau(n) = 6$ then $n = pq^2$ for distinct primes p and q.