Math 2065
Review Exercises for Exam II (With Answers)

The syllabus for Exam II is Sections 3.2 – 3.6 and 3.8 of Chapter 3 and Sections 4.1 and 4.3 of Chapter 4. You should review all of the assigned exercises in these sections. In addition, Section 3.1 contains the main formulas for computation of Laplace transforms, and hence, while not explicitly part of the exam syllabus, it will be necessary to be completely conversant with the computational techniques of Section 3.1. For this reason, some of the exercises related to this section from the last review sheet are repeated here. A Laplace transform table and table of convolution products will be provided with the test. Following is a brief list of terms, skills, and formulas with which you should be familiar.

- Know how to use all of the Laplace transform formulas developed in Section 3.1 to be able to compute the Laplace transform of elementary functions.

- Know how to use partial fraction decompositions to be able to compute the inverse Laplace transform of any proper rational function. The key recursion algorithms for computing partial fraction decompositions are Lemma 3 (Page 100) for the case of a real root in the denominator, and Lemma 1 (Page 110) for a complex root in the denominator. Here are the two results:

  Theorem 1 (Real Root Partial Fraction). *Let $P_0(s)$ and $Q(s)$ be polynomials. Assume that a is a number such that $Q(a) \neq 0$ and n is a positive integer. Then there is a unique number A_1 and polynomial $P_1(s)$ such that*

  $$\frac{P_0(s)}{(s-a)^n Q(s)} = \frac{A_1}{(s-a)^n} + \frac{P_1(s)}{(s-a)^{n-1}Q(s)}.$$

  The number A_1 and polynomial $P_1(s)$ are computed as follows:

  $$A_1 = \left.\frac{P_0(s)}{Q(s)}\right|_{s=a} \quad \text{and} \quad P_1(s) = \frac{P_0(s) - A_1 Q(s)}{s-a}.$$

  Theorem 2 (Complex Root Partial Fraction). *Let $P_0(s)$ and $Q(s)$ be polynomials. Assume that $a + bi$ is a complex number with nonzero imaginary part $b \neq 0$ such that $Q(a + bi) \neq 0$ and n is a positive integer. Then there is a unique linear term $B_1 s + C_1$ and polynomial $P_1(s)$ such that*

  $$\frac{P_0(s)}{((s-a)^2 + b^2)^n Q(s)} = \frac{B_1 s + C_1}{((s-a)^2 + b^2)^n} + \frac{P_1(s)}{((s-a)^2 + b^2)^{n-1} Q(s)}.$$

  The linear term $B_1 s + C_1$ and polynomial $P_1(s)$ are computed as follows:

  $$B_1 s + C_1|_{s=a+bi} = \left.\frac{P_0(s)}{Q(s)}\right|_{s=a+bi} \quad \text{and} \quad P_1(s) = \frac{P_0(s) - (B_1 s + C_1)Q(s)}{(s-a)^2 + b^2}.$$

- Know how to apply the basic differentiation formula (Theorem 6, Page 133) to compute the solution of an initial value problem for a constant coefficient linear differential equation with elementary forcing function.

- Know how to use the *characteristic polynomial* to be able to solve constant coefficient homogeneous linear differential equations. (See Algorithm 3, Page 181.)

The following is a small set of exercises of types identical to those already assigned.

1. (a) Complete the following definition: Suppose $f(t)$ is a continuous function of exponential type defined for all $t \geq 0$. The **Laplace transform** of f is the function $F(s)$ defined as follows

$$F(s) = \mathcal{L}(f(t))(s) = \boxed{\int_0^\infty e^{-st} f(t)\,dt}$$

for all s sufficiently large.

(b) Using your definition compute the Laplace transform of the function $f(t) = 2t - 5$. You may need to review the integration by parts formula: $\int u\,dv = uv - \int v\,du$.

▶ **Solution.** The Laplace transform of $f(t) = 2t - 5$ is the integral

$$\mathcal{L}(2t-5)(s) = \int_0^\infty (2t-5)e^{-st}\,dt,$$

which is computed using the integration by parts formula by letting $u = 2t - 5$ and $dv = e^{-wt}\,dt$, so that $du = 2\,dt$ while $v = -\frac{1}{s}e^{-st}$. Thus, if $s > 0$,

$$\begin{aligned}
\mathcal{L}(2t-5)(s) &= \int_0^\infty (2t-5)e^{-st}\,dt \\
&= -\frac{2t-5}{s}e^{-st}\Big|_0^\infty + \int_0^\infty \frac{2}{s}e^{-st}\,dt \\
&= \left(-\frac{2t-5}{s}e^{-st} - \frac{2}{s^2}e^{-st}\right)\Big|_0^\infty \\
&= -\frac{5}{s} + \frac{2}{s^2}.
\end{aligned}$$

The last evaluation uses the fact (verified in calculus) that $\lim_{t\to\infty} e^{-st} = 0$ and $\lim_{t\to\infty} te^{-st} = 0$ provided $s > 0$. ◀

2. Compute the Laplace transform of each of the following functions using the Laplace transform Tables (See Pages 158–161). (A Laplace Transform tables will be provided to you on the exam.)

(a) $f(t) = 3t^3 - 2t^2 + 7$

$$\boxed{F(s) = 3\frac{3!}{s^4} - 2\frac{2!}{s^3} + \frac{7}{s} = \frac{18}{s^4} - \frac{4}{s^3} + \frac{7}{s}.}$$

(b) $g(t) = e^{-3t} + \sin\sqrt{2}t$

$$\boxed{G(s) = \frac{1}{s+3} + \frac{\sqrt{2}}{s^2+2}.}$$

(c) $h(t) = -8 + \cos(t/2)$

$$\boxed{H(s) = -\frac{8}{s} + \frac{2}{s^2+1/4} = -\frac{8}{s} + \frac{4s}{4s^2+1}.}$$

3. Compute the Laplace transform of each of the following functions. You may use the Laplace Transform Tables.

(a) $f(t) = 7e^{2t}\cos 3t - 2e^{7t}\sin 5t$

$$\boxed{F(s) = \frac{7s}{(s-2)^2+9} - \frac{10}{(s-7)^2+25}.}$$

(b) $g(t) = 3t\sin 2t$

▶ **Solution.** Use formula 21, Page 160: $\mathcal{L}\{tf(t)\}(s) = -F'(s)$. Apply this formula to the function $f(t) = 3\sin 2t$ so that $F(s) = 6/(s^2+4)$. Since $g(t) = tf(t)$, formula 21) gives:

$$G(s) = -F'(s) = -\frac{-12s}{(s^2+4)^2} = \frac{12s}{(s^2+4)^2}.$$

◀

(c) $h(t) = (2-t^2)e^{-5t}$

▶ **Solution.** Use the shifting theorem (Formula 18, Page 160). Then

$$\boxed{H(s) = \frac{2}{s+5} - \frac{2}{(s+5)^3}.}$$

◀

4. Find the inverse Laplace transform of each of the following functions. You may use the Laplace Transform Tables.

(a) $F(s) = \dfrac{7}{(s+3)^3}$

$$\boxed{f(t) = \frac{7}{2}t^2e^{-3t}.}$$

(b) $G(s) = \dfrac{s+2}{s^2-3s-4}$

▶ **Solution.** Use partial fractions to write

$$G(s) = \frac{s+2}{s^2-3s-4} = \frac{1}{5}\left(\frac{6}{s-4} - \frac{1}{s+1}\right).$$

Thus $\boxed{g(t) = \left(6e^{4t} - e^{-t}\right)/5}$. ◀

(c) $H(s) = \dfrac{s}{(s+4)^2+4}$

▶ **Solution.** Since

$$H(s) = \frac{s}{(s+4)^2+4} = \frac{(s+4)-4}{(s+4)^2+4} = \frac{s+4}{(s+4)^2+4} - 2\frac{2}{(s+4)^2+4},$$

it follows that $\boxed{h(t) = e^{-4t}\cos 2t - 2e^{-4t}\sin 2t}$. ◀

5. Find the Laplace transform of each of the following functions.

(a) t^2e^{-9t}

$$\boxed{\frac{2}{(s+9)^3}}$$

(b) $e^{2t} - t^3 + t^2 - \sin 5t$

$$\boxed{\frac{1}{s-2} - \frac{6}{s^4} + \frac{2}{s^3} - \frac{5}{s^2+25}}$$

(c) $t\cos 6t$

$$\boxed{-\frac{d}{ds}\left(\frac{s}{s^2+36}\right) = \frac{s^2-36}{(s^2+36)^2}}$$

(d) $2\sin t + 3\cos 2t$

$$\boxed{\frac{2}{s^2+1} + \frac{3s}{s^2+4}}$$

(e) $e^{-5t}\sin 6t$

$$\boxed{\frac{6}{(s+5)^2+36}}$$

(f) $t^2\cos at$ where a is a constant

▶ **Solution.** Use Formula 22, Page 160, applied to $f(t) = \cos at$. Then, $F(s) = s/(s^2+a^2)$ and $\mathcal{L}\left\{t^2\cos at\right\}(s) = F''(s)$. Since $F'(s) = (a^2-s^2)/(s^2+a^2)^2$, the Laplace transform of $t^2\cos at$ is

$$\boxed{F''(s) = \frac{2s^2-6sa^2}{(s^2+a^2)^3}.}$$

◀

6. Find the inverse Laplace transform of each of the following functions.

(a) $\dfrac{1}{s^2-10s+9}$

▶ **Solution.** Since $s^2-10s-9 = (s-9)(s-1)$, use partial fractions:

$$\frac{1}{s^2-10s+9} = \frac{1}{8}\left(\frac{1}{s-9} - \frac{1}{s-1}\right) \Rightarrow \boxed{\mathcal{L}^{-1}\left\{\frac{1}{s^2-10s+9}\right\} = \frac{1}{8}(e^{9t}-e^{t}).}$$

◀

(b) $\dfrac{2s-18}{s^2+9}$ $\boxed{2\cos 3t - 6\sin 3t}$

(c) $\dfrac{2s+18}{s^2+25}$ $\boxed{2\cos 5t + (18/5)\sin 5t}$

(d) $\dfrac{s+3}{s^2+5}$ $\boxed{\cos\sqrt{5}t + (3/\sqrt{5})\sin\sqrt{5}t}$

(e) $\dfrac{s-3}{s^2-6s+25}$

▶ **Solution.** Since $s^2-6s+25 = (s-3)^2+4^2$, we conclude:

$$\boxed{\mathcal{L}^{-1}\left\{\frac{s-3}{s^2-6s+25}\right\} = e^{3t}\cos 4t.}$$

◀

(f) $\dfrac{1}{s(s^2+4)}$

▶ **Solution.** Use Formula 25, Page 160, with $F(s) = 1/(s^2+4)$ so that $f(t) = \sin 2t$. Then

$$\boxed{\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+4)}\right\} = \int_0^t \sin 2v\, dv = \frac{1}{2}\left(1 - \cos 2t\right).}$$

◀

(g) $\dfrac{1}{s^2(s+1)^2}$

▶ **Solution.** Use Formula 25 (Page 160) twice, starting with $F(s) = 1/(s+1)^2$ (so $f(t) = te^{-t}$). Using the integral formula $\int ue^{au}\, du = \frac{1}{a^2}(au-1)e^{au} + C$ (which can be found in a table of integrals or derived by integration by parts, we find using Formula 25:

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)^2}\right\} = \int_0^t ve^{-v}\, dv = (-v-1)e^{-v}\Big|_0^t = -te^{-t} - e^{-t} + 1.$$

Now integrating the right hand side a second time from 0 to t gives (after some algebraic simplification:

$$\boxed{\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\} = -te^{-t} + 2e^{-t} + t - 2.}$$

This exercise can also be solved by partial fraction expansion and by use of the convolution product formula. ◀

7. Compute the Laplace transform of each of the following functions.

(a) $f(t) = 3(e^t)^4 + \sin\sqrt{2}t$ $\quad \boxed{F(s) = \dfrac{3}{s-4} + \dfrac{\sqrt{2}}{s^2+2}}$

(b) $g(t) = 5t^3 - 3\cos 5t + \frac{3}{5}$ $\quad \boxed{G(s) = \dfrac{30}{s^4} - \dfrac{3s}{s^2+25} + \dfrac{3}{5s}}$

8. Solve each of the following differential equations by means of the Laplace transform:

(a) $y' + 3y = t^2e^{-3t} + te^{-2t} + t, \qquad y(0) = 1$

▶ **Solution.** If $Y(s) = \mathcal{L}(y(t))$ then applying $\mathcal{L}$ to both sides of the equation gives:

$$sY(s) - 1 + 3Y(s) = \frac{2}{(s+3)^3} + \frac{1}{(s+2)^2} + \frac{1}{s^2};$$

and solving for $Y(s)$:

$$Y(s) = \frac{1}{s+3} + \frac{2}{(s+3)^4} + \frac{1}{(s+3)(s+2)^2} + \frac{1}{s^2(s+3)}.$$

Using partial fractions:

$$\frac{1}{(s+3)(s+2)^2} = \frac{1}{s+3} - \frac{1}{s+2} + \frac{1}{(s+2)^2}$$
$$\frac{1}{s^2(s+3)} = \frac{1}{9}\left(\frac{1}{s+3} - \frac{1}{s} + \frac{3}{s^2}\right)$$

Therefore, combining like terms in $Y(s)$ gives

$$Y(s) = \frac{18}{9(s+3)} + \frac{2}{(s+3)^4} - \frac{1}{s+2} + \frac{1}{(s+2)^2} - \frac{1}{9s} + \frac{1}{3s^2},$$

which gives

$$\boxed{y(t) = \frac{18}{9}e^{-3t} + \frac{1}{3}t^3e^{-3t} - e^{-2t} + te^{-2t} - \frac{1}{9} + \frac{t}{3}.}$$

◀

(b) $y'' - 3y' + 2y = 4, \qquad y(0) = 2, \quad y'(0) = 3$

▶ **Solution.** As usual, $Y = \mathcal{L}(y)$. Applying $\mathcal{L}$ to both sides of the equation gives

$$s^2Y(s) - 2s - 3 - 3(Y(s) - 2) + 2Y(s) = \frac{4}{s}$$

and solving for $Y(s)$ gives:

$$\begin{aligned} Y(s) &= \frac{2s^2 - 3s + 4}{s(s-2)(s-1)} \\ &= \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-2} \\ &= \frac{2}{s} - \frac{3}{s-1} + \frac{3}{s-2}, \end{aligned}$$

where the last two lines represent the decomposition of $Y(s)$ into partial fractions. Taking the inverse Laplace transform gives

$$\boxed{y(t) = 2 - 3e^t + 3e^{2t}.}$$

◀

(c) $y'' + 4y = 6\sin t, \qquad y(0) = 6, \quad y'(0) = 0$

▶ **Solution.** As usual, $Y = \mathcal{L}(y)$. Applying $\mathcal{L}$ to both sides of the equation and solving for Y gives:

$$Y(s) = \frac{6s}{s^2+4} + \frac{6}{(s^2+4)(s^2+1)} = \frac{6s}{s^2+4} + \frac{2}{s^2+1} - \frac{2}{s^2+4}.$$

Taking the inverse Laplace transform gives

$$\boxed{y(t) = 6\cos 2t + 2\sin t - \sin 2t.}$$

◀

(d) $y''' - y' = 2, \qquad y(0) = y'(0) = y''(0) = 4$

▶ **Solution.** Letting $Y(s) = \mathcal{L}(y(t))$ and taking the Laplace transform of both sides of the equation gives:

$$s^3Y(s) - 4s^2 - 4s - 4 - (sY(s) - 4) = \frac{2}{s};$$

solving for $Y(s)$ and expanding in partial fractions gives

$$\begin{aligned} Y(s) &= \frac{4s^3 + 4s^2 + 2}{s^4 - s^2} \\ &= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s-1} \\ &= \frac{-2}{s^2} - \frac{1}{s+1} + \frac{5}{s-1}. \end{aligned}$$

Now taking the inverse Laplace transform gives

$$\boxed{y(t) = 5e^t - e^{-t} - 2t.}$$

◀

(e) $y''' - y' = 6 - 3t^2, \qquad y(0) = y'(0) = y''(0) = 1 \qquad \boxed{y(t) = t^3 + e^t}$

9. Using the Laplace transform, find the solution of the following differential equations with initial conditions $y(0) = 0$, $y'(0) = 0$:

(a) $y'' - y = 2\sin t \quad \boxed{y(t) = (1/2)(e^t - e^{-t}) - \sin t}$

(b) $y'' + 2y' = 5y \quad \boxed{y(t) = 0}$

(c) $y'' + y = \sin 4t \quad \boxed{y(t) = (1/15)(4\sin t - \sin 4t)}$

(d) $y'' + y' = 1 + 2t \quad \boxed{y(t) = 1 - e^{-t} + t^2 - t}$

(e) $y'' + 4y' + 3y = 6 \quad \boxed{y(t) = e^{-3t} - 3e^{-t} + 2}$

(f) $y'' - 2y' = 3(t + e^{2t}) \quad \boxed{y(t) = (3/8)(1 - 2t - 2t^2 - e^{2t} + 4te^{2t}}$

(g) $y'' - 2y' = 20e^{-t}\cos t \quad \boxed{y(t) = 3e^{2t} - 5 + 2e^{-t}(\cos t - 2\sin t)}$

(h) $y'' + y = 2 + 2\cos t$ $\boxed{y(t) = 2 - 2\cos t + t\sin t}$

(i) $y'' - y' = 30\cos 3t$ $\boxed{y(t) = 3e^t - 3\cos 3t - \sin 3t}$

10. Compute the convolution $t * t^3$ directly from the definition.

► **Solution.**

$$\begin{aligned}
t * t^3 &= \int_0^t \tau(t-\tau)^3\, d\tau \\
&= \int_0^t \tau(t^3 - 3t^2\tau + 3t\tau^2 - \tau^3)\, d\tau \\
&= \int_0^t \left(t^3\tau - 3t^2\tau^2 + 3t\tau^3 - \tau^4\right)\, d\tau \\
&= \left(t^3\frac{\tau^2}{2} - t^2\tau^3 + \frac{3}{4}t\tau^4 - \frac{\tau^5}{5}\right)\Big|_0^t \\
&= t^5\left(\frac{1}{2} - 1 + \frac{3}{4} - \frac{1}{5}\right) \\
&= \frac{t^5}{20}.
\end{aligned}$$

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11. Using the table of convolutions, compute each of the following convolutions:

(a) $(1+3t) * e^{5t}$

► **Solution.**

$$\begin{aligned}
(1+3t) * e^{5t} &= 1 * e^{5t} + 3t * e^{5t} \\
&= \int_0^t e^{5\tau}\, d\tau + 3\left(\frac{e^{5t} - (1+5t)}{25}\right) \\
&= \frac{1}{5}(e^{5t} - 1) + 3\left(\frac{e^{5t} - (1+5t)}{25}\right) \\
&= \frac{8e^{5t} - 8 - 15t}{25}.
\end{aligned}$$

◄

(b) $(1/2 + 2t^2) * \cos\sqrt{2}t$ $\boxed{2t - \frac{3\sqrt{2}}{4}\sin\sqrt{2}t}$

(c) $(e^{2t} - 3e^{4t}) * (e^{2t} + 4e^{3t})$ $\boxed{te^{2t} - \frac{5}{2}e^{2t} + 16e^{3t} - \frac{27}{2}e^{4t}}$

12. Solve each of the following homogeneous linear differential equations, using the techniques of Chapter 4 (Characteristic and/or indicial equation).

(a) $y'' + 3y' + 2y = 0$

(b) $y'' + 6y' + 13y = 0$

(c) $y'' + 6y' + 9y = 0$

(d) $y'' - 2y' - y = 0$

(e) $8y'' + 4y' + y = 0$

(f) $2y'' - 7y' + 5y = 0$

(g) $2y'' + 2y' + y = 0$

(h) $y'' + .2y' + .01y = 0$

(i) $y'' + 7y' + 12y = 0$

(j) $y'' + 2y' + 2y = 0$

(k) $y''' + 2y'' - 15y' = 0$

(l) $y''' + 2y'' - 8y' = 0$

(m) $y''' - 2y'' - 3y' = 0$

(n) $y''' - 7y' + 6y = 0$

(o) $y''' - 3y'' - y' + 3y = 0$

(p) $4y''' - 10y' + 12y = 0$

(q) $y^{(4)} - 5y'' + 4y = 0$

Answers

(a) $y = c_1 e^{-t} + c_2 e^{-2t}$

(b) $y = c_1 e^{-3t} \cos 2t + c_2 e^{-3t} \sin 2t$

(c) $y = (c_1 + c_2 t) e^{-3t}$

(d) $y = c_1 e^{(1+\sqrt{2})t} + c_2 e^{(1-\sqrt{2})t}$

(e) $y = e^{-t/4}(c_1 \cos \frac{t}{4} + c_2 \sin \frac{t}{4})$

(f) $y = c_1 e^{5t/2} + c_2 e^{t}$

(g) $y = e^{-t/2}(c_1 \cos(-t/2) + c_2 \sin(t/2))$

(h) $y = e^{0.1t}(c_1 + tc_2)$

(i) $y = c_1 e^{-4t} + c_2 e^{-3t}$

(j) $y = e^{-t}(c_1 \cos t + c_2 \sin t)$

(k) $y = c_1 + c_2 e^{3t} + c_3 e^{-5t}$

(l) $y = c_1 + c_2 e^{2t} + c_3 e^{-4t}$

(m) $y = c_1 + c_2 e^{-t} + c_3 e^{3t}$

(n) $y = c_2 e^t + c_2 e^{2t} + c_3 e^{-3t}$

(o) $y = c_1 e^{3t} + c_2 e^t + c_3 e^{-t}$

(p) $y = c_1 e^{-2t} + c_2 e^{t/2} + c_3 e^{3t/2}$

(q) $y = c_1 e^{2t} + c_2 e^{-2t} + c_3 e^t + c_4 e^{-t}$

13. Find the general solution of the constant coefficient homogeneous linear differential equation with the given characteristic polynomial $p(s)$.

(a) $p(s) = (s-1)(s+3)(s-5)$

(b) $p(s) = s^3 - 1$

(c) $p(s) = (s^2 - 2)^2$

(d) $p(s) = s^3 - 3s^2 + s + 5$

(e) $p(s) = s^4 + 3s^2 - 4$

(f) $p(s) = s^4 + 5s^2 + 4$

(g) $p(s) = (s^2 + 1)^3$

(h) $p(s)$ has degree 4 and has roots $\sqrt{2}$ with multiplicity 2 and $2 \pm 3i$ with multiplicity 1.

(i) $p(s)$ has degree 5 and roots 0 with multiplicity 3 and $1 \pm \sqrt{3}$ with multiplicity 1.

(j) $p(s)$ has degree 5 and roots 0 with multiplicity 3 and $1 \pm \sqrt{3}i$ with multiplicity 1.

Answers

(a) $y = c_1 e^t + c_2 e^{-3t} + c_3 e^{5t}$

(b) $y = c_1 e^t + c_2 e^{-t/2} \cos \sqrt{3}t/2 + c_2 e^{-t/2} \sin \sqrt{3}t/2$

(c) $y = (c_1 + c_2 t) e^{\sqrt{2}t} + (c_3 + c_4 t) e^{-\sqrt{2}t}$

(d) $y = c_1 e^{-t} + c_2 e^{2t} \cos t + c_3 e^{2t} \sin t$

(e) $y = c_1 \cos 2t + c_2 \sin 2t + c_3 e^t + c_4 e^{-t}$

(f) $y = c_1 \cos 2t + c_2 \sin 2t + c_3 \cos t + c_4 \sin t$

(g) $y = (c_1 + c_2 t + c_3 t^2) \cos t + (c_4 + c_5 t + c_6 t^2) \sin t$

(h) $y = (c_1 + c_2 t) e^{\sqrt{2}t} + c_3 e^{2t} \cos 3t + c_4 e^{2t} \sin 3t$

(i) $y = (c_1 + c_2 t + c_3 t^2) + c_4 e^{(1+\sqrt{3})t} + c_5 e^{(1-\sqrt{3})t}$

(j) $y = (c_1 + c_2 t + c_3 t^2) + c_4 e^t \cos \sqrt{3}t + c_5 e^t \sin \sqrt{3}t$

14. Solve each of the following initial value problems. You may (and should) use the work already done in exercise 12.

(a) $y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = -3.$

(b) $y'' + 6y' + 13y = 0, \quad y(0) = 0, \quad y'(0) = -1.$

(c) $y'' + 6y' + 9y = 0, \quad y(0) = -1, \quad y'(0) = 5.$

(d) $y'' - 2y' - y = 0, \quad y(0) = 0, \quad y'(0) = \sqrt{2}.$

(e) $y'' + 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 2$

Answers

(a) $y = 2e^{-2t} - e^{-t}$

(b) $y = -\frac{1}{2}e^{-3t}\sin 2t$

(c) $y = (-1 + 2t)e^{-3t}$

(d) $y = \frac{1}{2}(e^{(1+\sqrt{2})t} - e^{(1-\sqrt{2})t})$

(e) $y = 2e^{-t}\sin t$

15. Find a second order linear homogeneous differential equation with constant real coefficients that has the given function as a solution, or explain why there is no such equation.

(a) $e^{-3t} + 2e^{-t}$

(b) $e^{-t}\cos 2t$

(c) $e^t t^{-2}$

(d) $5e^{3t/2} + 7e^{-t}$

(e) $2e^{3t}\sin(t/2) - (1/2)e^{3t}\cos(t/2)$

Answers

(a) $y'' + 4y' + 3y = 0$

(b) $y'' + 2y' + 5y = 0$

(c) Not possible since $e^t t^{-2}$ is not included in the list of functions in Theorem 3.3.1 (Page 146).

(d) $2y'' - y' - 3y = 0$

(e) $4y'' - 24y' + 37y = 0$