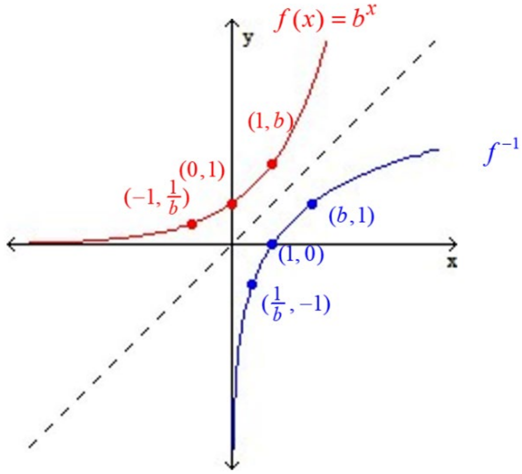
Section 5.2 Logarithmic Functions

Objective 1: Understanding the Definition of a Logarithmic Function

Every exponential function of the form $f(x)=b^x$, where $b>0$ and $b\neq 1$, is one-to-one and thus has an inverse function. The graph of $f(x)=b^x, b>1$ and its inverse, f^{-1}, are shown below. Recall from Section 5.1, the graph of $f(x)=b^x, b>1$ contains the points $\left(-1,\dfrac{1}{b}\right)$, $(0,1)$ and $(1,b)$, and since $b^x \to 0$ as $x\to -\infty$, the x-axis is a horizontal asymptote for the graph. Recall from Section 3.6 that the graph of f^{-1} is obtained by reflecting the graph of *f* about the line $y=x$. Therefore, the graph of f^{-1} will contain the points $\left(\dfrac{1}{b},-1\right)$, $(1,0)$ and $(b,1)$, and the y-axis will be a vertical asymptote for the graph.



To find the equation of f^{-1}, we begin with the process from Section 3.6:

Step 1: Change $f(x)$ to y: $y=b^x$
Step 2: Interchange x and y: $x=b^y$
Step 3: Solve for y: ??
Before we can solve for y, we must introduce the following definition:

Definition: For $x>0$, $b>0$ and $b\neq 1$, the **logarithmic function with base *b*** is defined by $y=\log_b x$ if and only if $x=b^y$.

Step 3. **Solve for y:** $x=b^y$ can be written as $y=\log_b x$
Step 4. **Change y to $f^{-1}(x)$:** $f^{-1}(x)=\log_b x$

Objective 2: Evaluating Logarithmic Expressions

The expression $\log_b x$ is the exponent to which b must be raised to in order to get x.

Objective 3: Understanding the Properties of Logarithms

General Properties of Logarithms

For $b > 0$ and $b \neq 1$,

(1) $\log_b b = 1$ and

(2) $\log_b 1 = 0$.

Cancellation Properties of Exponentials and Logarithms

For $b > 0$ and $b \neq 1$,

(1) $b^{\log_b x} = x$ and

(2) $\log_b b^x = x$.

Objective 4: Using the Common and Natural Logarithms

Definition: For $x > 0,$ the **common logarithmic function** is defined by $y = \log x$ if and only if $x = 10^y$.

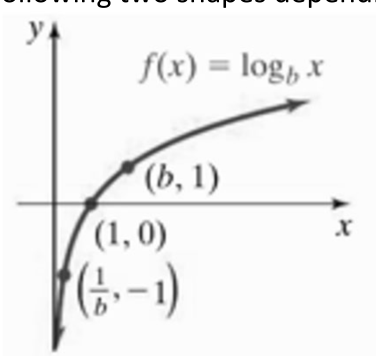
Definition: For $x > 0,$ the **natural logarithmic function** is defined by $y = \ln x$ if and only if $x = e^y$.

Objective 5: Understanding the Characteristics of Logarithmic Functions

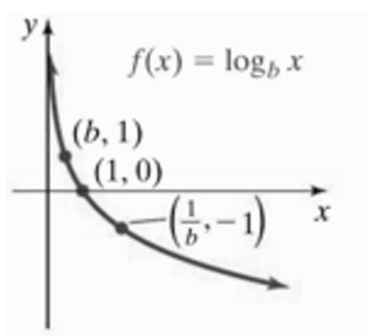
Characteristics of Logarithmic Functions

For $b > 0$ and $b \neq 1$, the logarithmic function with base b is defined by $y = \log_b x$.

The domain of $f(x) = \log_b x$ is $(0, \infty)$ and the range is $(-\infty, \infty)$. The graph of $f(x) = \log_b x$ has one of the following two shapes depending on the value of b:



$f(x) = \log_b x$, $b > 1$



$f(x) = \log_b x$, $0 < b < 1$

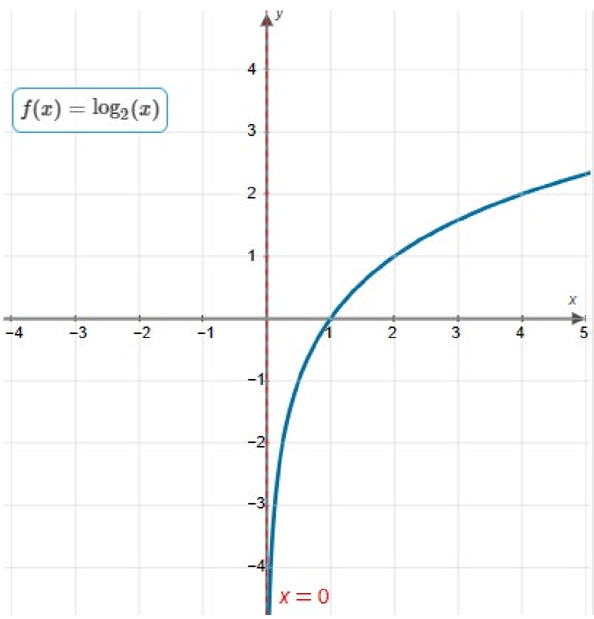
The graph of $y = \log_b x$, $b > 0$ and $b \neq 1$ has the following properties:

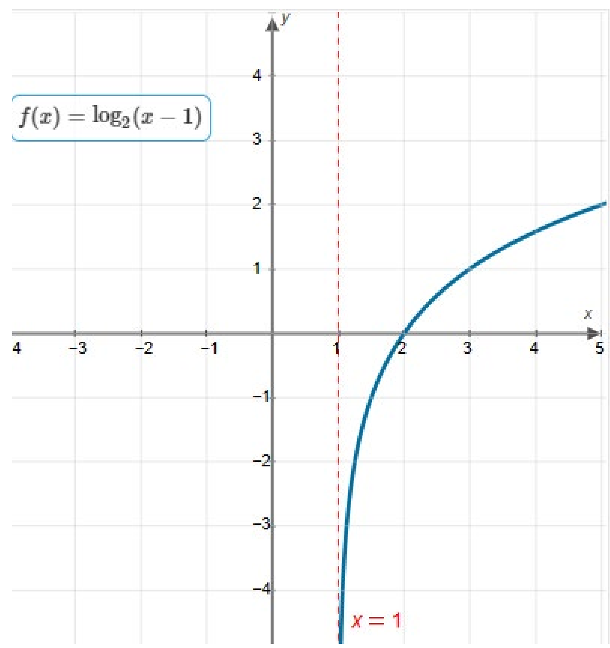
1. The graph intersects the x-axis at $(1,0)$.
2. The graph contains the points $(b,1)$ and $\left(\frac{1}{b}, -1\right)$.
3. If $b > 1$, the graph is increasing on the interval $(0, \infty)$.

   If $0 < b < 1$, the graph is decreasing on the interval $(0, \infty)$.
4. The y-axis ($x = 0$) is a vertical asymptote.
5. The function is one-to-one.

Objective 6: Sketching the Graphs of Logarithmic Functions Using Transformations

The graph of $f(x)=\log_2(x-1)$ can be obtained by horizontally shifting the graph of $f(x)=\log_2 x$ to the right one unit. The graph of the function $f(x)=\log_2 x$ is shown below on the left. It contains the points $\left(\frac{1}{2},-1\right)$, $(1,0)$ and $(2,1)$ and has vertical asymptote $x=0$. To shift the graph of this function right one unit, add 1 to each of the x-coordinates of the points on the graph. The resulting graph of $f(x)=\log_2(x-1)$, shown below on the right, contains the points $\left(\frac{3}{2},-1\right)$, $(2,0)$ and $(3,1)$ and has vertical asymptote $x=1$. The domain of $f(x)=\log_2(x-1)$ is $(1,\infty)$ and the range is $(-\infty,\infty)$.





Objective 7: Finding the Domain of Logarithmic Functions

If $f(x) = \log_b \left[g(x) \right]$, then the domain of *f* can be found by solving the inequality $g(x) > 0$.