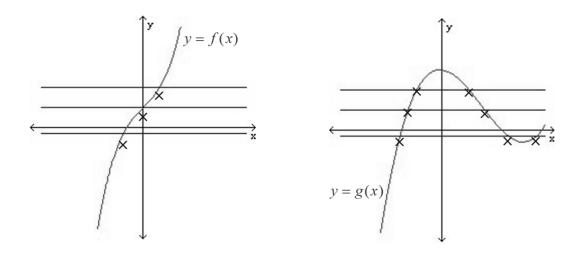
Section 3.6 One-to-one Functions; Inverse Functions

Objective 1: Understanding the Definition of a One-to-one Function

Definition: A function f is **one-to-one** if for any values $a \neq b$ in the domain of f, $f(a) \neq f(b)$.

Interpretation: For f(x) = y to be a function, we know that for each x in the domain there exists one and only one y in the range. For f(x) = y to be a one-to-one function, both of the following must be true: for each x in the domain there exists one and only one y in the range, AND for each yin the range there exists one and only one x in the domain.





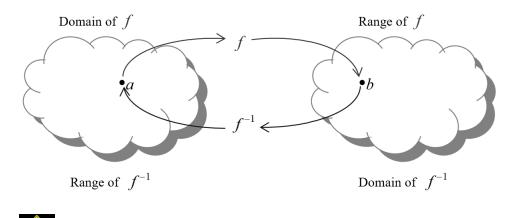
The Horizontal Line Test

If every horizontal line intersects the graph of a function *f* at most once, then *f* is one-to-one.

Objective 3: Understanding and Verifying Inverse Functions

Every one-to-one function has an inverse function.

Definition: Let f be a one-to-one function with domain A and range B. Then f^{-1} is **the inverse** function of f with domain B and range A. Furthermore, if f(a) = b then $f^{-1}(b) = a$.



Do not confuse f^{-1} with $\frac{1}{f(x)}$. The negative 1 in f^{-1} is NOT an exponent!

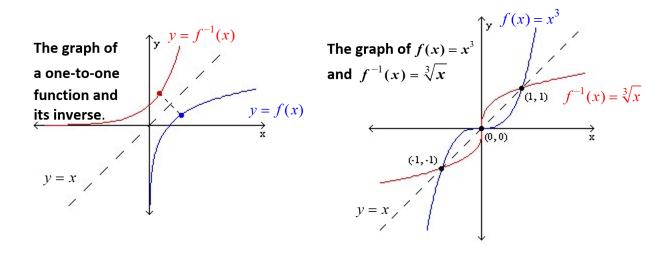
Inverse functions "undo" each other.

Composition Cancellation Equations:

 $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1} $f^{-1}(f(x)) = x$ for all x in the domain of f

Objective 4: Sketching the Graphs of Inverse Functions

The graph of f^{-1} is a reflection of the graph of f about the line y = x. If the functions have any points in common, they must lie along the line y = x.



Objective 5: Finding the Inverse of a One-to-one Function

We know that if a point (x, y) is on the graph of a one-to-one function, then the point (y, x) is on the graph of its inverse function.

To find the inverse of a one-to-one function, replace f(x) with y, interchange the variables x and y, and then solve for y. This is the function $f^{-1}(x)$.

Inverse Function Summary

- 1. The inverse function f^{-1} exists if and only if the function f is one-to-one.
- 2. The domain of f is the same as the range of f^{-1} and the range of f is the same as the domain of f^{-1} .
- 3. To verify that two one-to-one functions f and g are inverses of each other, use the composition cancellation equations to show that f(g(x)) = g(f(x)) = x.
- 4. The graph of f^{-1} is a reflection of the graph of f about the line y = x. That is, for any point (a,b) that lies on the graph of f, the point (b,a) must lie on the graph of f^{-1} .
- 5. To find the inverse of a one-to-one function, replace f(x) with y, interchange the variables x and y, and then solve for y. This is the function $f^{-1}(x)$.