**Section 3.4 Transformations of Functions**

# Objective 1: Using Vertical Shifts to Graph Functions

Let *c* be a positive real number.

1. The graph of  is obtained by shifting the graph of  vertically upward *c* units.
2. The graph of  is obtained by shifting the graph of  vertically downward *c* units.

# Objective 2: Using Horizontal Shifts to Graph Functions

Let *c* be a positive real number.

1. The graph of  is obtained by shifting the graph of  horizontally to the left *c* units.
2. The graph of  is obtained by shifting the graph of  horizontally to the right *c* units.

For , the graph of is the graph of *f* shifted to the **right** *c* units. At first glance, it appears that the rule for horizontal shifts is the opposite of what seems natural. Substituting  for *x* causes the graph of to be shifted to the left while substituting  for *x* causes the graph to shift to the right *c* units.

# Objective 3: Using Reflections to Graph Functions

The graph of  is a **reflection** of the graph of  **about the *x-*axis**.

The graph of  is a **reflection** of the graph of  **about the *y-*axis**.

# Objective 4: Using Vertical Stretches and Compressions to Graph Functions

Suppose *a* is a positive real number. The graph of  is obtained by the multiplying each *y-*coordinate of  by *a*.

1. If , the graph of is a **vertical stretch** of the graph of .
2. If , the graph of  is a **vertical compression** of the graph of .

# Objective 6: Using Combinations of Transformations to Graph Functions

When graphing a function that involves multiple transformations, it is important to follow a certain “order of operations.” In our text, transformations are performed in the following order:

1. Horizontal shifts
2. Reflection about *y*-axis
3. Vertical stretches/compressions
4. Reflection about *x*-axis
5. Vertical shifts

Although different ordering is possible, the order above will always work.

# Objective 7: Using Transformations to Sketch the Graphs of Piecewise-Defined Functions