Section 2.3 Lines

Objective 1: Determining the Slope of a Line

In mathematics, the steepness of a line can be measured by computing the line's **slope.** Every non-vertical line has slope. Vertical lines are said to have **no slope** (or an **undefined slope**).

A line going up from left to right has **positive slope**, a line going down from left to right has **negative slope,** and a horizontal line has **zero slope**. We use the variable m to represent slope.

The slope can be computed by comparing the vertical change (the **rise)** to the horizontal change (the **run**). Given any two points on the line, the slope m can be computed by taking the quotient of the rise over the run.

Definition: If $x_1 \neq x_2$, the **slope** of a line passing through distinct points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{rise}{run} = \frac{Change\,in\,y}{Change\,in\,x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Objective 2: Sketching a Line Given a Point and the Slope

If we know the slope of the line and any point that lies on the graph of the line, then we can quickly sketch the line.

Objective 3: Finding the Equation of a Line Using the Point-Slope Form

The **Point-Slope Form of the Equation of a Line** passing through the point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$.

Objective 4: Finding the Equation of a Line Using the Slope-Intercept Equation

The slope-intercept form of the equation of a line is extremely important since every non-vertical line has exactly one slope-intercept equation.

The **Slope-Intercept Form of the Equation of a Line** with slope m and y-intercept b is given by $y = mx + b$.

Objective 5: Writing the Equation of a Line in Standard Form

The Standard Form of an Equation of a Line is given by $Ax + By = C$ where A, B, and C are real numbers such that A and B are not both zero.

Note that every equation of a line in two variables can be written in standard form. Furthermore, if the coefficients are rational, then fractions can always be eliminated by multiplying both sides of the equation by the least common denominator. Therefore, the standard form of the equation of a line seen in this etext and in the exercises will always include non-fractional coefficients and A will always be greater than or equal to zero.

Objective 6: Finding the Slope and the *y*-intercept of a Line in Standard Form

Suppose we are given the standard form of the equation of a line $Ax + By = C$ with $B \neq 0$ and wish to solve for y. To do this, we subtract Ax from both sides and divide by *B* to obtain $y = -\frac{A}{B}x + \frac{C}{B}$ which is the equation of the line in slope-intercept form. Thus, given the standard form of the equation of a line $Ax + By = C$ with $B \neq 0$, the slope of the line is $m = -\frac{A}{B}$ and the y-intercept is $b = \frac{C}{B}$.

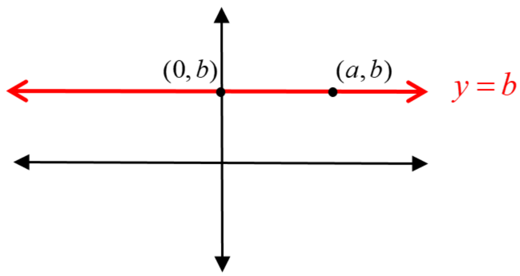
Objective 7: Sketching Lines by Plotting Intercepts

The x-intercept is found by setting $y = 0$ and solving for x.
The y-intercept is found by setting $x = 0$ and solving for y.

Objective 8: Finding the Equation of a Horizontal Line and a Vertical Line

Horizontal Lines

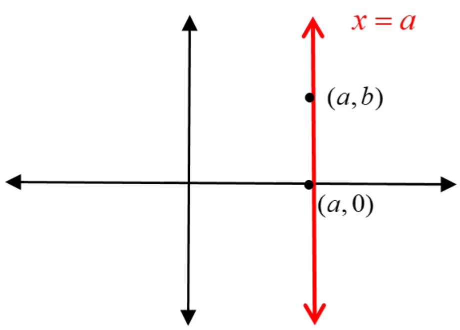
Suppose we wish to determine the equation of the horizontal line that contains the point (a,b). To find this equation we must first determine the slope. Since the line must also pass through the point $(0,b)$, we see that the slope of this line is $m=\frac{b-b}{a-0}=\frac{0}{a}=0$. Using the slope-intercept form of a line with $m=0$ and y-intercept b, we see that the equation is $y=0x+b$ or $y=b$.



Therefore, we know that for any horizontal line that contains the point (a,b), the equation of that line is $y=b$ and the slope is $m=0$.

Vertical Lines

Vertical lines have **no slope** or **undefined slope**. We can see this by looking at the vertical line that passes through the point (a,b). Because this line also passes through the x-intercept at the point $(a,0)$, we see that the slope of this line is $m=\frac{b-0}{a-a}=\frac{b}{0}$ which is not a real number since division by zero is not defined. Since the x-coordinate of this vertical line is always equal to a regardless of the y-coordinate, we say that the equation of a vertical line is $x=a$.



Therefore, we know that for any vertical line that contains the point (a,b), the equation of that line is $x=a$, and the slope is undefined.

SUMMARY OF FORMS OF EQUATIONS OF LINES

Point-Slope Form: $y - y_1 = m(x - x_1)$

Slope is m and (x_1, y_1) is a point on the line.

Standard Form: $Ax + By = C$

A, B, and C are real numbers with A and B not both 0 and $A \geq 0$.

Horizontal Line: $y = b$

Slope is 0 and y-intercept is b.

Vertical Line: $x = a$

Undefined slope and x-intercept is a.