Section 1.1a Linear and Rational Equations

Objective 1: Recognizing Linear Equations

A linear equation in one variable involves variables that are only raised to the first power.

Definition: A **linear equation in one variable** is an equation that can be written in the form ax + b = c, where a, b, and c are real numbers and $a \ne 0$.

Examples of linear equations:

$$\frac{7}{2}x - 4 = x \qquad \sqrt{5}x - 3 = 8x - 1 \qquad (0.7)^2 x + 1 = 6 \qquad 2^{-1} - 0.4x = 0.3 - 5x$$

Examples of non-linear equations:

$$\frac{3}{x} - 5 = 9 \qquad 5\sqrt{x} + 1 = 7 \qquad x^2 + x = 6 \qquad (0.7x)^2 + x = 1 \qquad 0.3 - 0.2x^{-1} = 2$$

Objective 2: Solving Linear Equations with Integer Coefficients

When we solve an equation for x, we are looking for all values of x which, when substituted back into the original equation, yield a true statement. The goal here is to **isolate the variable** x on one side of the equation.

Objective 3: Solving Linear Equations Involving Fractions

One way to solve a linear equation involving fractions is to first transform the equation into a linear equation involving integer coefficients. We can accomplish this using the multiplication property of equality by multiplying both sides of the equation by the lease common denominator (LCD).

Objective 4: Solving Linear Equations Involving Decimals

The strategy for solving linear equations involving decimals is similar to the one used to solve linear equations involving fractions. We want to eliminate all decimals by multiplying both sides of the equation by the smallest power of 10 (such as $10^1 = 10$, $10^2 = 100$, etc) that will guarantee that the new linear equation will not include decimals. To determine the smallest power of 10, look at all terms in the equation that contain a decimal factor and choose the factor that has the greatest number of decimal places. Count these decimal places and then raise 10 to that power.