Section 9.2 and 9.4

Simplifying Square Roots and Rationalizing Denominators

# Objective 1: Simplifying Square Roots Using the Product Rule

A square root is simplified when the radicand contains no perfect square factors other than $1$. For example, $\sqrt{20}$ is not simplified because $\sqrt{20}=\sqrt{4⋅5}$ and $4$ is a perfect square.

**Product Rule for Square Roots:**

If $\sqrt{a}$ and $\sqrt{b}$ are real numbers, then

$\sqrt{a⋅b}=\sqrt{a}⋅\sqrt{b}$.

Applying this rule, we can simplify $\sqrt{20}$ as follows:

$$\sqrt{20}=\sqrt{4⋅5}=\sqrt{4}⋅\sqrt{5}=2\sqrt{5}$$

Simplify.

|  |  |
| --- | --- |
| a. $\sqrt{54}$ | b. $\sqrt{500}$ |
| c. $3\sqrt{12}$ |  |

# Objective 2: Simplifying Square Roots Using the Quotient Rule

Next, we will examine the square root of a quotient.

**Quotient Rule for Square Roots:**

If $\sqrt{a}$ and $\sqrt{b}$ are real numbers and $b\ne 0$, then

$\sqrt{\frac{a}{b}} =\frac{\sqrt{a}}{\sqrt{b}}$.

Simplify.

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| --- | --- |
| a. $\sqrt{\frac{5}{49}}$ | b. $\sqrt{\frac{1}{121}}$ |

# Objective 3: Rationalizing Denominators

It is sometimes easier to work with a radical expression if the denominator does not contain a radical. Rewriting a radical expression to eliminate a radical in the denominator is called **rationalizing** the denominator.

For example, the expression $\frac{\sqrt{5}}{\sqrt{2}}$ has an irrational numerator and denominator. We can rationalize the denominator as follows:

$\frac{\sqrt{5}}{\sqrt{2}}=\frac{\sqrt{5}}{\sqrt{2}}⋅\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{5}⋅\sqrt{2}}{\sqrt{2}⋅\sqrt{2}}=\frac{\sqrt{5⋅2}}{\sqrt{2⋅2}}=\frac{\sqrt{10}}{\sqrt{4}}=\frac{\sqrt{10}}{2}$

The expression $\frac{\sqrt{10}}{2}$ is numerically equivalent to the original expression but has a rational denominator.

Rationalize the denominator and simplify.

|  |  |
| --- | --- |
| a. $\frac{\sqrt{13}}{\sqrt{15}}$ | b. $\frac{1}{\sqrt{72}}$ |