Section 6.2 Adding and Subtracting Polynomials

# Objective 1: Defining Polynomial

In this section, we will explore a type of algebraic expression called a **polynomial expression**.

**Polynomial**

A polynomial in $x$ is a finite sum of terms of the form $ax^{n}$ where $a$ is a real number and $n$ is a whole number.

For example,

$$x^{3}-2x^{2}+3x+5$$

is a polynomial. Notice that this polynomial is written in descending powers of $x$. When the terms of a polynomial expression are arranged in this way, it is called the **standard form** of a polynomial expression.

The **coefficient** is the numerical factor of each term. If a term is a number only, it is called a **constant** term, or just constant. The polynomial shown above has four **terms**:

$x^{3}$ which has a coefficient of $1$

$-2x^{2}$ which has a coefficient of $-2$

$3x$ which has a coefficient of $3$

$5$ which is the constant.

The expression $4x^{-2}+8x-9$ is not a polynomial because it contains an exponent that is not a whole number.

Each term of a polynomial has a **degree**. The degree of a term is the sum of the exponents on the variables contained in the term.

The **degree of a polynomial** is the greatest degree of any term of the polynomial.

Determine if the expression is a polynomial. If it is a polynomial, state the degree.

|  |  |
| --- | --- |
| a. $x^{2}-8x^{5}+6x^{3}+9x-5$ | b. $x+x^{\frac{1}{2}}-6$ |

A polynomial with exactly one term is called a **monomial**. A polynomial with exactly two terms is called a **binomial**. A polynomial with exactly three terms is called a **trinomial**.

Polynomials can contain more than one variable as long as all exponents on the variables are whole numbers.

Determine the degree of the polynomial expression. Then state if the polynomial is a monomial, binomial, trinomial, or none of these.

|  |  |
| --- | --- |
| c. $-2x^{2}y+7xy^{3}$ | d. $3a^{2}+2ab+b^{2}$ |

# Objective 2: Simplifying Polynomials by Combining Like Terms

Polynomials that contain like terms can be simplified by combining the like terms. Recall that like terms are terms that contain exactly the same variables raised to exactly the same powers. For example, $5a^{2}b$ and $-2a^{2}b$ are like terms, but $5a^{2}b$ and $-2ab^{2}$ are unlike terms.

Simplify the polynomial by combining like terms.

|  |  |
| --- | --- |
| a. $3b^{2}-8b^{2}+12b$ | b. $\frac{1}{2}xy+3x^{2}y^{2}+\frac{3}{2}xy-x^{2}y^{2}$ |

# Objective 3: Adding and Subtracting Polynomials

To add polynomials, combine all like terms.

Add the polynomials.

|  |  |
| --- | --- |
| a. $\left(3x^{2}+8x-5\right)+\left(2x^{2}-6\right)$ | b. $\left(6a-7b+3\right)+(2a+9b+5)$ |

To subtract one polynomial from another, we add its opposite.

Subtract the polynomials.

|  |  |
| --- | --- |
| c. $\left(9x^{2}+4y^{2}\right)-\left(-2x^{2}+10y^{2}\right)$ | d. $\left(9t^{3}+t^{2}-12t-4\right)-(t^{3}-t^{2}+15t)$ |

Simplify the expression.

e. $\left(5c-4d\right)+\left(4c+8d\right)-(2d-c)$