Section 4.1 Linear Inequalities and Problem Solving

# Objective 1: Graphing Solution Sets to Linear Inequalities

A **linear inequality in one variable** is an inequality that can be written in the form

$$ax+b<c$$

where $a, b, $and $c$ are real numbers and $a$ is not $0$.

This definition and all other definitions, properties, and steps in this section are still valid for the inequality symbols $>$, $\geq $, or $\leq $.

A **solution of an inequality** is a value of the variable that makes the inequality a true statement. The **solution set** is the set of all solutions.

For example, the inequality $x<3$ is true when $x$ is replaced with any number less than $3$, or put another way, for any number to the left of $3$ on the number line. Since there are infinitely many solutions, we cannot list them all. We use set builder notation to write the solutions.

$$\{x|x<3\}$$

This is read “The set of all $x$ such that $x$ is less than $3$.”

We can also graph the solution set on the number line.



Graph $x\geq 5$ on the number line.



# Objective 2: Solving Linear Inequalities

The process of solving a linear inequality is similar to the process used to solve a linear equation. Our goal is to isolate the variable by using properties of inequality.

**Addition Property of Inequality:**

If $a$, $b$, and $c$ are real numbers, then

$a<b$ and $a+c<b+c$

are equivalent inequalities.

a. Solve the inequality $x-7\leq -10$. Graph the solution set.



**Multiplication Property of Inequality:**

1. If $a$, $b$, and $c$ are real numbers and $c$ is positive, then

$a<b$ and $ac<bc$

are equivalent inequalities.

1. If $a$, $b$, and $c$ are real numbers and $c$ is negative, then

$a<b$ and $ac>bc$

are equivalent inequalities.

If we multiply or divide both sides of an inequality by a negative number, the direction of the inequality sign must be reversed for the inequalities to remain equivalent.

Solve the inequality. Graph the solution set.

b. $\frac{3}{8}x>-2$



c. $4x+5<10x+17$



d. $3(5-x)\leq \frac{21}{2}$



# Objective 3: Solving Linear Inequality Applications

Problems containing words such as “at least,” “at most,” “no more than,” or “no less than” indicate that an inequality may be useful in solving the problem.

Ben bowled $125$ and $183$ in his first two games. What must he bowl in his third game to have an average of at least $180?$