Section 3.4 Slope and Rate of Change

# Objective 1: Finding the Slope of a Line Given Two Points of the Line

The slant or steepness of a line is formally known as its **slope**. We measure the slope of a line by the ratio of vertical change to the corresponding horizontal change as we move along the line.

For example, consider the line graphed below. We can select any two points that lie on the line and measure the vertical change (which is the change in the $y$-coordinates) and the horizontal change (which is the change in the $x$-coordinates). The ratio of these changes is the slope of the line.



$$Slope=\frac{change in y}{change in x}=\frac{6-2}{4-1}=\frac{4}{3}$$

The slope of this line is $\frac{4}{3}$ which means that for every $4$ units of change in the $y$-coordinates there is a corresponding change of $3$ units in the $x$-coordinates. Note that we could have selected any two points that lie on this line, and the slope calculation would always be $\frac{4}{3}$.

To find the slope of a line, choose two points on the line which we will denote as $(x\_{1},y\_{1})$ and $(x\_{2}, y\_{2})$. The vertical change, or **rise**, between these points is the difference in the $y$-coordinates. The horizontal change, or **run**, between these points is the difference of the $x$-coordinates.



**Slope of a Line:**

The slope $m$ of the line containing the points $(x\_{1},y\_{1})$ and $(x\_{2}, y\_{2})$ is given by

$m=\frac{rise}{run}=\frac{change in y}{change in x}=\frac{y\_{2}-y\_{1}}{x\_{2}-x\_{1}}$, as long as $x\_{2}\ne x\_{1}$.

Find the slope of the line that goes through the given points.

|  |  |
| --- | --- |
| a. $(-4,-1)$ and $(5, -13)$ | b. $(8, -2)$ and $(3,-4)$ |

If a line rises to the right, then the line has a positive slope. If the line falls to the right, then the line has a negative slope.

Find the slope of the line.

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| --- | --- |
| c. Line with a point at (0,1) and a point at (3,5). |  |

# Objective 2: Finding the Slope of a Line Given Its Equation

One way to find the slope of a line given its equation is to write the equation in the form $y=mx+b$. This form is called the **slope-intercept form** because $m$ is the slope of the line and the point $(0,b)$ is the $y$-intercept of the line.

Find the slope and $y$-intercept of the line. Then graph the line.

|  |  |
| --- | --- |
| a. $y=6x-5$Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. | b. $5x-3y=0$Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. |
| c. $12y=24-3x$Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. |  |

# Objective 3: Finding Slopes of Vertical and Horizontal Lines

Graph the line. Then select two points on the line and find the slope by using the slope formula.

|  |  |
| --- | --- |
| a. $y=-1$Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. | b. $x=3$Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. |

Any two points on a horizontal line will have the same $y$-coordinates. This means that the change in the $y$-values will always be $0$. Thus, all horizontal lines have a slope of $0$.

Any two points on a vertical line will have the same $x$-coordinates. This means that the change in the $x$-values will always be $0$. Thus, all vertical lines have undefined slope.

# Objective 4: Slope as a Rate of Change

# Slope can be interpreted as a rate of change. In other words, slope tells us how fast $y$ is changing with respect to $x$. Many real world situations can be modeled using a linear equation.

The population of New Orleans in the year 2000 was $484,668$. Its population in 2020 was $383,997$.

a. Write two ordered pairs of the form (year, population).

b. Find the slope of the line through the two points.

c. Write a sentence explaining the meaning of the slope in this context.