Section 10.4b Further Graphing of Quadratic Functions

# Objective 1: Writing Quadratic Functions in the Form $f\left(x\right)=a(x-h)^{2}+k$

When the equation of a quadratic function is written in the form $f\left(x\right)=a\left(x-h\right)^{2}+k$, we can easily identify the vertex $(h,k)$ of its graph. When a quadratic function is given in the form $f\left(x\right)=ax^{2}+bx+c$, we can use the process of completing the square to rewrite it in the form $f\left(x\right)=a\left(x-h\right)^{2}+k$.

1. Consider the function $f\left(x\right)=x^{2}-6x+8$.

a. Rewrite the function in the form $f\left(x\right)=a\left(x-h\right)^{2}+k$.

b. Does $f$ have a maximum value or a minimum value? What is that value?

c. State the vertex and the axis of symmetry of the graph of $f$.

d. Find the $x$- and $y$-intercepts of the graph of $f$.

e. Graph $f$ showing the vertex, axis of symmetry, and intercepts. State the domain and range of $f$.



2. Consider the function $f\left(x\right)=6x^{2}+12x+5$.

a. Rewrite the function in the form $f\left(x\right)=a\left(x-h\right)^{2}+k$.

b. Does $f$ have a maximum value or a minimum value? What is that value?

c. State the vertex and the axis of symmetry of the graph of $f$.

d. Find the $x$- and $y$-intercepts of the graph of $f$.

e. Graph $f$ showing the vertex, axis of symmetry, and intercepts. State the domain and range of $f$.



3. Consider the function $f\left(x\right)=-x^{2}+4x-8$.

a. Rewrite the function in the form $f\left(x\right)=a\left(x-h\right)^{2}+k$.

b. Does $f$ have a maximum value or a minimum value? What is that value?

c. State the vertex and the axis of symmetry of the graph of $f$.

d. Find the $x$- and $y$-intercepts of the graph of $f$.

e. Graph $f$ showing the vertex, axis of symmetry, and intercepts. State the domain and range of $f$.



# Objective 2: Using the Vertex Formula

When given the equation of a quadratic function written in the form $f\left(x\right)=ax^{2}+bx+c$, there is also a formula we can use to find the vertex. One way to think about the formula is to consider the graph of a quadratic function that has two $x$-intercepts. We can find the $x$-intercepts by using the quadratic formula.



Using the symmetry of the graph of the parabola, we can find the $x$-coordinate of the vertex by averaging the values of the $x$-intercepts. Doing that, we find that the $x$-coordinate of the vertex is $-\frac{b}{2a}$. We can find the $y$-coordinate of the vertex by substituting the $x$-coordinate into the equation of the function.

**Vertex Formula**

The graph of the quadratic function $f\left(x\right)=ax^{2}+bx+c$ is a parabola with vertex $\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right)$.

1. Consider the function $f\left(x\right)=-2x^{2}-8x-8$.

a. Find the vertex and the axis of symmetry of the graph of $f$.

b. Find the $x$- and $y$-intercepts of the graph of $f$.

c. Graph $f$ showing the vertex, axis of symmetry, and intercepts. State the domain and range of $f$.



2. Consider the function $f\left(x\right)=\frac{1}{3}x^{2}+x$.

a. Find the vertex and the axis of symmetry of the graph of $f$.

b. Find the $x$- and $y$-intercepts of the graph of $f$.

c. Graph $f$ showing the vertex, axis of symmetry, and intercepts. State the domain and range of $f$.

